(1) Let $\Gamma$ be a circle with center $O$ and radius $r$ and let $A$ be a point in the exterior of $\Gamma$. Let $M$ be a point on $\Gamma$ and let $N$ be the point on $\Gamma$ such that $MN$ is a diameter. Determine the locus of the centers of the circles which pass through $A$, $M$, and $N$ as one varies $M$.

(2) Let $\triangle ABC$ be an obtuse angled triangle and let $A'$, $B'$, and $C'$ (respectively) be the points of intersection of the interior angle bisectors of angles $A$, $B$, and $C$ (respectively) with the opposite sides of the triangle. Now let:
- $A''$ be the intersection of $BC$ with the perpendicular bisector of $AA'$;
- $B''$ be the intersection of $AC$ with the perpendicular bisector of $BB'$;
- $C''$ be the intersection of $AB$ with the perpendicular bisector of $CC'$.

Show that $A''$, $B''$, and $C''$ are collinear.

(3) Let $O$ be the circumcenter of an acute angled triangle $ABC$ and $A_1$ a point on the arc $BC$ which is part of the circumcircle of the triangle $ABC$. Let $A_2$ and $A_3$ be points on the sides $AB$ and $AC$ respectively, such that $\angle BA_1A_2 = \angle OAC$ and $\angle CA_1A_3 = \angle OAB$. Show that the line segment $A_2A_3$ passes through the orthocenter of the triangle $ABC$.

(4) Given a set $S$ of points in the plane, we call a circle in the plane a 4-circle if it passes through at least four points of $S$. What is the maximum number of 4-circles that could be determined by a set of 7 points?

(5) We assign a real number between 0 and 1 to every point of the plane with integer coordinates. This is done in such a way that the number assigned to a given point is equal to the arithmetic mean of the numbers assigned to the four points that have distance one to the given point (the points directly above, below, to the left, and to the right of the given point). Show that all the numbers are equal.

(6) Determine the smallest real number $r$ such that it is possible to cover an equilateral triangle with side length 1 by six circles with radius $r$.

(7) Let $ABC$ be an equilateral triangle and $P$ an interior point such that $\angle APC = 120°$. Let $M$ be the intersection of $CP$ with $AB$ and $N$ be the intersection of $AP$ with $BC$. Find the locus of the circumcenter of the triangle $MBN$ as we vary $P$.

(8) Given a circle $\Gamma$, consider a quadrilateral $ABCD$ with its four sides tangent to $\Gamma$. Let $AD$ be tangent to $\Gamma$ at $P$ and $CD$

be tangent to $\Gamma$ at $Q$. Let $X$ and $Y$ be the points where the segment $BD$ intersects $\Gamma$, and let $M$ be the midpoint of $XY$. Show that $\angle AMP = \angle CMQ$.

(9) Let $M$ and $N$ be points on the sides $AC$ and $BC$ (respectively) of a triangle $ABC$, and let $P$ be a point on the line segment $MN$. Show that at least one of the triangles $AMP$ and $BNP$ has an area which is less than or equal to $\frac{1}{8}$ of the area of the triangle $ABC$.

(10) Let $ABCD$ be a convex quadrilateral. The extensions of $AB$ and $CD$ intersect in $E$, and the extensions of $AD$ and $BC$ intersect in $F$. The angle bisectors of $\angle A$ and $\angle C$ intersect in $P$, and the angle bisector of $\angle B$ and $\angle D$ intersect in $Q$. The angle bisectors of the exterior angles at $E$ (for triangle $ADE$) and $F$ (for triangle $ABF$) intersect in $R$. Show that $P$, $Q$, and $R$ are collinear.