

MAT 3321, COMPLEX ANALYSIS AND INTEGRAL TRANSFORMS,  
WINTER 2005

Answers to Homework 7  
14.1 #14; 14.2 #2,4

**Problem 14.1 #14** To show that the series  $\sum_{n=0}^{\infty} \frac{i^n}{n^2+i}$  is convergent, we use the comparison test. As discussed in class, the series  $\sum_{n=0}^{\infty} \frac{1}{n^2}$  converges. We also have, for all  $n$ :

$$\left| \frac{i^n}{n^2+i} \right| = \frac{1}{\sqrt{n^4+1^2}} \leq \frac{1}{\sqrt{n^4}} = \frac{1}{n^2}$$

Therefore, by the comparison test, the first series converges too.

**Problem 14.2 #2** The power series  $\sum_{n=0}^{\infty} \frac{2^{20n}}{n!} (z-3)^n$  is in powers of  $z-3$ , therefore the center is at  $z_0 = 3$ . To determine the radius of convergence, we use the Cauchy-Hadamard formula:

$$\begin{aligned} R &= \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{2^{20n}}{n!}}{\frac{2^{20(n+1)}}{(n+1)!}} \right| = \lim_{n \rightarrow \infty} \left| \frac{2^{20n} (n+1)!}{n! 2^{20(n+1)}} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{2^{20n} (n+1)n!}{n! 2^{20n} 2^{20}} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)}{2^{20}} \right| = \infty \end{aligned}$$

Therefore, the series converges for all  $z$ .

**Problem 14.2 #4** The power series  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} \left(\frac{z}{4}\right)^{n+1} = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \frac{z^{n+1}}{4^{n+1}}$  is in powers of  $z$ , therefore the center is at  $z_0 = 0$ . We find the radius of convergence using the Cauchy-Hadamard formula.

$$\begin{aligned} R &= \lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{1}{n(n+1)4^{n+1}}}{\frac{1}{(n+1)(n+2)4^{n+2}}} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{4^{n+2}}{n(n+1)} \frac{(n+1)(n+2)}{4^{n+1}} \right| = 4. \end{aligned}$$