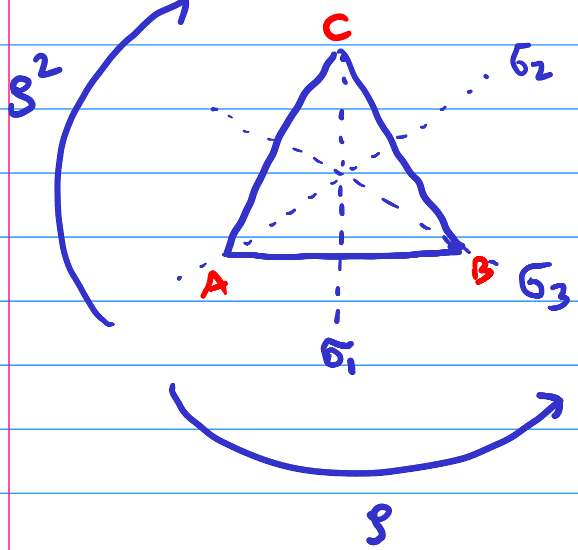
An equilateral triangle:



It has 6 symmetries.

σ_1

σ_2

σ_3

ρ 120° counterclockwise

ρ^2 240° " "

ε identity function.

They form a group. D_3

$$\rho^2 \circ \sigma_2 \circ \rho \circ \sigma_1 \circ \rho \stackrel{?}{=} \rho$$

<u>Def</u> Let G be a group, let X be a subset of G.

We say $\langle X \rangle$ is the smallest subgroup of G containing X.

Equivalently, $\langle X \rangle$ is the smallest subset of G such that:

(1) $\forall x \in X.\quad x \in \langle X \rangle$

(2) $\quad \varepsilon \in \langle X \rangle$

(3) $\forall v, w \in \langle X \rangle.\quad v \circ w \in \langle X \rangle$

(4) $\forall v \in \langle X \rangle.\quad v^{-1} \in \langle X \rangle$

We say that X is a set of generators for G if $\langle X \rangle = G$.

For D_3, $X = \{\rho, \sigma_1\}$ generates the group.

<u>Proof</u>

$$\varepsilon = \varepsilon$$
$$\rho = \rho$$
$$\rho^2 = \rho \circ \rho$$
$$\sigma_1 = \sigma_1$$
$$\sigma_2 = \rho \circ \rho \circ \sigma_1$$
$$\sigma_3 = \rho \circ \sigma_1$$

Also, $Y = \{\sigma_1, \sigma_2\}$ generates the group.

<u>Proof</u>

$$\varepsilon = \varepsilon$$
$$\rho = \sigma_1 \circ \sigma_2$$
$$\rho^2 = \sigma_2 \circ \sigma_1$$
$$\sigma_1 = \sigma_1$$
$$\sigma_2 = \sigma_2$$
$$\sigma_3 = \sigma_1 \circ \sigma_2 \circ \sigma_1$$

<u>Relations</u> $\sigma_2 = \boxed{\rho \circ \rho \circ \sigma_1 \sim \sigma_3 \circ \rho \circ \rho}$

A <u>relation</u> (an <u>equation</u>) is just a pair of words.

We say the relation <u>holds</u> in G if the LHS and RHS are equal.