MATH 316, DIFFERENTIAL EQUATIONS, WINTER 2000

Problem Set 1, due Wednesday, January 19

Problem 1 For each of the given (systems of) differential equations, determine what is its order and whether it is linear.

(a) $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 = x.$

(b) $\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \cos x.$

(c) $\begin{cases} \frac{d^2y}{dt^2} + t^2 z = 0 \\ \frac{dz}{dt} + t\frac{dy}{dt} = 1. \end{cases}$

(d) $\frac{d^2y}{dx^2} + \sin(\frac{dy}{dx}) = y.$

(e) $(\frac{dy}{dx})^2 - 3xy\frac{dy}{dx} + x^2y^2 = 0.$

Problem 2 Determine the values of r for which the given differential equation has solutions of the form $y = e^{rt}$.

(a) $y' + 2y = 0$

(b) $y'' - 2y' + y = 0$

(c) $y''' + y'' - 2y' = 0$

Problem 3 (a) Find all solutions to the differential equation $\frac{dy}{dt} + 3t^2y = t^2$. (b) Draw a sketch of the slope field and of the integral curves. Is there an equilibrium solution? If yes, is it stable or unstable? (c) Find the solution for which $y(0) = 1$.

Problem 4 (a) Draw a direction field for the differential equation $y' + y = 2te^{-t} + 3$. (b) Based on an inspection of the direction field, describe how solutions behave for large t. (c) Find the general solution and use it to determine how solutions behave as $t \to \infty$.

Problem 5 Consider the initial value problem

$$3y' - y = e^{t/5}, \qquad y(0) = a.$$

(a) Draw a direction field. How do solutions appear to behave as t becomes large? Does the behavior depend on the choice of the initial value a? Let a_0 be the value of a for which the transition from one type of behavior to another occurs. Estimate the value of a_0.

(b) Solve the initial value problem and find the critical value a_0 exactly.

(c) Describe the behavor of the solution corresponding to the initial value a_0.

Problem 6 Find the solution of

$$\frac{dy}{dt} = \frac{1}{e^y - t}, \qquad y(1) = 0.$$

Hint: Consider t as the dependent variable instead of y.

Problem 7 Do Problems 2.2 #15, 16 from the textbook.