**MATH 316, DIFFERENTIAL EQUATIONS, WINTER 2000**

Problem Set 10, due Friday, April 14

Problem 1 Find the location and type of all equilibrium points and draw a phase portrait of the following non-linear differential equations:

$$(a) \quad \begin{aligned} x' &= y(x^2 - 4) \\ y' &= x(1 - y^2) \end{aligned} \qquad (b) \quad \begin{aligned} x' &= xy \\ y' &= 1 - x^2 - y^2 \end{aligned} \qquad (c) \quad \begin{aligned} x' &= \sin(3x + y) \\ y' &= 3\sin(\tfrac{x}{3} + y) \end{aligned}$$

Problem 2 Consider the following simple spring pendulum:
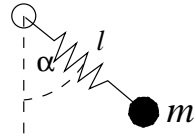
The mass is m, and the spring constant is k. The relaxed length of the spring is l_0. This is a 2-dimensional system. Let $q_0 = \alpha$ be the angle between the pendulum and a vertical line, as in the illustration, and let $q_1 = l$ be the length of the arm of the pendulum.

(a) Find formulas for the potential and kinetic energy in terms of q_0, q_1, $\dot{q}_0$, and $\dot{q}_1$.

(b) Use the Lagrangian formula to find the equation of motion for this system.