MATH 316, DIFFERENTIAL EQUATIONS, WINTER 2000

Problem Set 7 (and practice for 2nd exam), due Wednesday, March 22

Problem 1

(a) Find the general solution to the system of two differential equations below.

$$\left.\begin{array}{l} \frac{dx}{dt} = 2y - 3x \\ \frac{dy}{dt} = 5x + 6y \end{array}\right\}$$

(b) Draw a sketch of the solutions to the system in part (a) in the (x, y)-plane, indicating clearly in what direction t is increasing along your curves, as well as the behaviour as $t \to \infty$.

(c) Find the particular solution of the system in part (a) for which $x(0) = -1$ and $y(0) = 6$ and indicate its position on your sketch from (b).

Problem 2 Consider the differential equation

$$\frac{d^3y}{dx^3} + p(x)\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} + r(x)y = s(x),$$

for the function $y = y(x)$.

(a) Write down an equivalent system of first order linear differential equations.

(b) State the existence and uniqueness theorem for this differential equation. including all the assumptions and the full statement of the result.

(c) Solve the equation $y''' + 3y'' - 4y = 0$ with $y(0) = 3$, $y'(0) = 0$ and $y''(0) = -3$.

Problem 3

(a) Find values of n for which $x = t^n$ is a solution of the equation

$$t^2\frac{d^2x}{dt^2} + 3t\frac{dx}{dt} + 2x = 0.$$

(b) Hence write down the general solution to the equation

$$t^2\frac{d^2x}{dt^2} + 3t\frac{dx}{dt} + 2x = 10.$$

(c) Find the particular solution to the equation in part (b) for which $x(1) = 7$ and $\dot{x}(1) = -2$. Where is this solution defined?