MATH 2135, LINEAR ALGEBRA, Winter 2013

Handout 4: Problems
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Problem 1.

$$A = \begin{pmatrix} 1 & 2 & -3 & 4 & 5 \\ 2 & 3 & -1 & 5 & 7 \\ 1 & 0 & -3 & 3 & 4 \\ 2 & 1 & -2 & 4 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 1 & -3 \\ 1 & 1 & -3 & 1 \\ 1 & -3 & 1 & 1 \\ -3 & 1 & 1 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ 2 & 4 & 1 & 3 & 1 \\ 3 & 6 & 2 & 4 & 1 \\ 1 & 2 & 1 & 1 & 0 \end{pmatrix}$$

For each matrix A, B, C: (a) Find its row canonical form. (b) Find the rank. (c) Find a basis for the null space. (d) Find the columns that are linear combinations of preceding columns (and express each of them explicitly as a linear combination of preceding columns). (e) Find a basis for the column space. (f) Find the rows that are linear combinations of preceding rows. (g) Find a basis of the row space.

Problem 2. Determine which of the following matrices have the same row space:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 4 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 5 & 2 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & 3 & -2 \\ 1 & 1 & -2 \end{pmatrix}$$

Problem 3. Consider the following matrix with scalars in $\mathbb{Z}_2$:

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

(a) List all elements of the null space. (b) Find a basis of the null space. (c) List all element of the row space. (d) Find a basis of the row space.

Problem 4. On $\mathbb{R}^3$, consider the bases

$$S = \{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}\}, \quad T = \{\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}\}.$$

(a) For each of the following vectors v_i, find its coordinates $[v_i]_S$ in basis S, and its coordinates $[v_i]_T$ in basis T.

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}.$$

(b) There is a vector v such that $[v]_T = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$. Find $[v]_S$.

Problem 5. Let $V = P_3(t)$, and consider the map $f : V \to V$ such that for every polynomial $p \in P_3(t)$, $f(p) = p'$, where p' is the derivative of p. Let $S = \{1, t, t^2, t^3\}$. Find the matrix representation $[f]_S$ of f.

Problem 6. Consider the map $f : \mathbb{R}^3 \to \mathbb{R}^4$ given by $f(v) = Av$, where

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 3 & 8 & 0 \\ 1 & 3 & 0 \\ 2 & 3 & -1 \end{pmatrix}.$$

Consider the basis $S = \{\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\}$ for $\mathbb{R}^3$, and the basis $T = \{\begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix}\}$ for $\mathbb{R}^4$. Find the matrix representation $[f]_{S,T}$ of f with respect to the bases S and T.