An Introduction to Quantum Programming in Quipper

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Abstract. Quipper is a recently developed programming language for expressing quantum computations. This paper gives a brief tutorial introduction to the language, through a demonstration of how to make use of some of its key features. We illustrate many of Quipper's language features by developing a few well known examples of Quantum computation, including quantum teleportation, the quantum Fourier transform, and a quantum circuit for addition.

Keywords: Quantum Computation, Programming Languages, Quipper

1 Introduction

1.1 Overview

Quipper [10] is an embedded functional programming language for quantum computation. It has been developed as part of IARPA's QCS project [13]. The stated goal of the QCS project is to "*accurately estimate and reduce the computational resources required to implement quantum algorithms on a realistic quantum computer*", with an emphasis on using techniques that have been developed in the realms of computer science.

In this paper, we will look at how Quipper can be used to implement existing quantum algorithms, through a close look at some of the language features that have been added specifically for this task. Quipper's development was guided by the goal of implementing seven non-trivial quantum algorithms from the literature [3,5,11,12,14,17,18]. These algorithms were chosen by the QCS project, and provided to us in modified form. They cover a broad spectrum of techniques used in quantum computation. Each algorithm introduced its own challenges that helped guide the language features that are now available in Quipper.

We will use simple examples to try to demonstrate the use of Quipper, and to give insights into the types of problems that the various language features are useful for. We will consider three main stand-alone examples:

– Quantum teleportation will guide us through: Quipper's underlying circuit model, Quipper's primitive operations, quantum data-types, generic functions, comments, and labels.
– The quantum Fourier transform and quantum addition will help us look at: recursion, circuit-level operators, boxed circuits, and simulation.
– We will end by looking at Quipper's features that can be used to implement quantum oracles, including: automatic generation of circuits from classical code, synthesis of reversible circuits, and circuit transformations.

We will also have a brief look at how Quipper can be used to estimate the computational resources required by the algorithms that have been implemented.

In another recent paper [10], we have described in more detail the rationale behind the various design choices that went into Quipper, including a high-level overview of, and justification for, its language features. We also gave more background on general issues affecting quantum programming languages, and on the implementation of the language itself. By contrast, the aim of this present paper is to give a tutorial introduction to Quipper from a programmer's perspective, using examples that have been chosen to guide readers through some of Quipper's main features.

1.2 Quipper as an embedded language

Quipper has been implemented as an embedded language, using Haskell as the host language. Therefore, Quipper can be seen as a collection of data types, combinators, and a library of functions within Haskell, together with an *idiom*, i.e., a preferred style of writing embedded programs. In this paper, we present Quipper as if it were a language in its own right, i.e., without presupposing any knowledge of Haskell.

While the embedded language approach has many advantages (see [6, Sec. 1.3] for a general discussion), there are also certain potential pitfalls that programmers should be aware of. One of these is the temptation to "escape to the host language", i.e., to write general Haskell programs rather than following Quipper's intended idiom. This can break intended abstractions, and make the programs less portable in case of implementation changes. Another drawback of the embedded language approach is that compilation errors are often difficult to decipher, because the compiler presents them in terms of concepts of the host language, rather than the embedded language. Finally, while Haskell is a good fit for Quipper in many respects, it does lack two features that would be useful for a quantum programming language: *linear types* and *dependent types*. We must therefore live with checking certain well-formedness properties of programs at run-time, although they could in principle be checked by the type-checker in a dedicated language.

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1.3 Quipper's underlying circuit model

Quipper uses an extended circuit model of quantum computation. We allow for both quantum and classical wires and operations within a circuit. Quantum operations can be controlled by a classical wire, but not vice versa. A quantum wire can be explicitly measured, thus creating a classical wire. Quipper's circuit model also incorporates explicitly scoped ancilla wires, allowing for an ancilla to only come into scope for the part of the circuit in which it is used. This is achieved by allowing explicit qubit initialization and termination within a circuit.

Using a circuit model leads to three distinct phases of execution: compile time, circuit generation time, and circuit execution time. This, in turn, gives rise to an extra distinction among inputs. Inputs whose value is known at circuit generation time will be called *parameters*; whereas inputs whose value is only known at circuit execution time will be called *inputs*. To keep this distinction explicit, Quipper introduces three basic types for bits and qubits. We use the type `Bool` for a boolean parameter that is known at circuit generation time, the type `Bit` for a classical boolean input to a circuit, and the type `Qubit` for a quantum input to a circuit. A parameter of type `Bool` can easily be converted to an input of type `Bit`, but not vice versa. Also, because measurements can only occur at circuit runtime, the outcome of a measurement is a `Bit`, not a `Bool`.

2 Quipper by example

2.1 Quantum teleportation

Quipper's primitive operations. Although Quipper can be regarded as a language for describing quantum circuits, when actually developing computations within Quipper it is often preferable to think in terms of gates being applied in real time to qubits (or bits) that are held in variables. This procedural paradigm is the foundation for developing quantum computations in Quipper, on top of which more powerful higher-order operators are built.

Computations in Quipper take the form of functions. The following example shows how we can write a simple quantum function in Quipper.

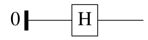
```
plus_minus :: Bool -> Circ Qubit
plus_minus b = do
    q <- qinit b
    r <- hadamard q
    return r
```

The first line corresponds to the *type* of the function. We see that the input to the function is a boolean parameter. The output type of the function is `Circ Qubit`. The `Circ` part of the type is actually a type operator, and is used to state that the function being defined can have a physical side effect when it is evaluated (Haskell programmers will recognize this as a *monad*). The `Qubit` part of the output type tells us that the function returns a qubit. The body of the function usually starts with the keyword `do`, followed by a block of quantum operations to be evaluated in the given order. The body of the `plus_minus` function uses three operations. The `qinit` operator initializes a new qubit, in the state corresponding to b. Here, `False` corresponds to $|0\rangle$ and `True` corresponds to $|1\rangle$. The notation tells us that this newly created qubit is stored in the variable q. The operator `hadamard` applies the Hadamard gate to the qubit q, storing the updated qubit in the variable r. The last line returns the qubit r as the output of the whole function. In summary, this function introduces a newly initialized qubit in either of the states $|+\rangle$ or $|-\rangle$ depending on a boolean parameter. We also note that variables in the function body are used *linearly*: each qubit is written exactly once and read exactly once. This restriction is imposed by the laws of quantum physics. In Quipper's syntax, however, it would have been permitted to use the same name for the two variables q and r, and we will often do so in future examples.

Circuit generation. After defining a quantum function in Quipper, there are various things we can do with it. The most basic of these is to evaluate the function to generate a circuit. When Quipper evaluates a circuit producing function, the circuit is produced lazily, on-the-fly. This is useful for defining very large circuits, whereby the whole circuit doesn't need to be stored in memory. Moreover, circuits can also be *consumed* lazily, for example by a transformation (see p. 13), or by passing instructions sequentially to an (actual or simulated) quantum computer (see p. 11).

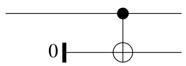
A useful operation provided by Quipper is a circuit printing function that enables the circuits produced by Quipper to be exported in various formats. For example, to produce a PDF document from the circuit defined by the above `plus_minus` function, we can use the built-in Quipper operator `print_simple`. Note that parameters, but not inputs, must be specified at circuit generation time. Here, we set the parameter b to `False`.

```
print_plus_minus :: IO ()
print_plus_minus = print_simple PDF (plus_minus False)
```



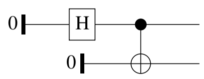
The circuit diagrams used throughout the rest of this paper have been created directly from the given code examples. The next example illustrates how to control a quantum gate. This function inputs a qubit and returns a pair of qubits. The `qnot` operation applies a not-gate to the qubit b. Moreover, the infix operator `‘controlled‘` causes this operation to be controlled by the qubit a. The overall effect of the function `share` is to take a qubit in the state $\alpha|0\rangle + \beta|1\rangle$ and entangle it with a newly initialized qubit to create the state $\alpha|00\rangle + \beta|11\rangle$.

```
share :: Qubit -> Circ (Qubit, Qubit)
share a = do
  b <- qinit False
  b <- qnot b ‘controlled‘ a
  return (a,b)
```



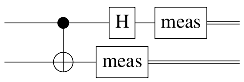
Previously defined quantum functions can be used as building blocks in other quantum functions. In fact, they can be used in exactly the same way as Quipper's built-in operators. In the next example, we use our previously defined functions, `plus_minus` and `share`, to produce a pair of qubits in the Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

```
bell00 :: Circ (Qubit, Qubit)
bell00 = do
  a <- plus_minus False
  (a,b) <- share a
  return (a,b)
```



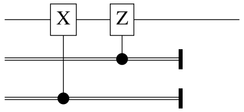
A teleportation circuit. Let us now consider quantum teleportation (see [15] for an introduction). This involves two parties Alice and Bob. Alice's goal is to teleport a qubit q to Bob. Alice and Bob must each have access to a single qubit from an entangled Bell pair (a, b), which we can produce with the above `bell00` function. We can think of Alice's role in terms of a function that inputs the two qubits q and a. The output of the function will be a pair of classical bits, produced by Alice by applying some unitary gates and then measuring both qubits.

```
alice :: Qubit -> Qubit -> Circ (Bit,Bit)
alice q a = do
    a <- qnot a `controlled` q
    q <- hadamard q
    (x,y) <- measure (q,a)
    return (x,y)
```



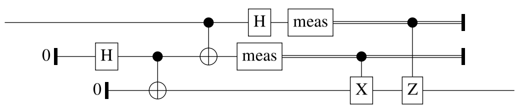
Note that the function `measure` has been applied to a pair of qubits. In Quipper's syntax, this is simply an abbreviation for measuring both qubits in the pair. This abbreviated syntax is possible because the Quipper operator `measure` is a *generic* operator: it can be applied to any data structure containing qubits, and returns a corresponding data structure containing bits. Another example of a generic Quipper operator is `cdiscard`, which can be applied to any data structure containing classical bits. It is used in Bob's part of the teleportation protocol:

```
bob :: Qubit -> (Bit,Bit) -> Circ Qubit
bob b (x,y) = do
    b <- gate_X b `controlled` y
    b <- gate_Z b `controlled` x
    cdiscard (x,y)
    return b
```



The following function ties all the pieces of the teleportation example together. We can see that a Bell state is created, which is then used by Alice, along with the input qubit, to create a pair of classical bits. These are passed to Bob along with his qubit from the Bell state. The generated circuit diagram shows that Quipper joined together the various steps as expected.

```
teleport :: Qubit -> Circ Qubit
teleport q = do
    (a,b) <- bell00
    (x,y) <- alice q a
    b <- bob b (x,y)
    return b
```



Quantum data types and generic functions. *Quantum data types* are types that are built up from `Qubit` by means of data constructors, such as tuples and lists. For example, `(Qubit,[Qubit])` is the type whose elements are pairs of a qubit and a (variable but finite length) list of qubits. Every quantum data type, such as qa = `(Qubit,[Qubit])`, has an associated *classical data type*, such as ca = `(Bit,[Bit])`, and *boolean data type*, such as ba = `(Bool,[Bool])`. We say that qa, ca, and ba have the same *shape*, but different *leaf types*. A Quipper function is called *generic* if it can act on data types of any shape.

We have already seen several examples of generic built-in Quipper functions, namely `measure`, `cdiscard`, and `print_simple`. However, what makes generic functions particularly useful in Quipper is the fact that it is easy to create new *user-defined* generic functions. We will now illustrate this feature by defining a generic version of the teleportation circuit.

In Quipper, the keyword `QShape` is used to declare that three types qa, ca, and ba are the quantum, classical, and boolean version of some data type. To define a generic version of the `plus_minus` function, we replace `Bool` and `Qubit` in its type by such a pair of related ba and qa:

```
plus_minus_generic :: (QShape ba qa ca) => ba -> Circ qa
plus_minus_generic a = do
    qs <- qinit a
    qs <- mapUnary hadamard qs
    return qs
```

We note that the `qinit` function is already generic. The operator `mapUnary` maps a function of type `Qubit` $\to$ `Circ Qubit` over every qubit in a quantum data structure. To extend the `share` function, we use the function `qc_false` which generates a boolean data structure of the correct shape, with every boolean set to `False`. The `mapBinary` function is similar to `mapUnary`, but maps a function of the type `Qubit` $\to$ `Qubit` $\to$ `Circ`(`Qubit`,`Qubit`) over every corresponding pair of qubits from two quantum data structures of the same shape. We also use the built-in `controlled_not` operation.

```
share_generic :: (QShape a qa ca) => qa -> Circ (qa, qa)
share_generic qa = do
    qb <- qinit (qc_false qa)
    (qb, qa) <- mapBinary controlled_not qb qa
    return (qa, qb)
```

Updating the `bell00` function requires a little more thought, as we now need to explicitly know the shape of the data being teleported in order to generate enough Bell pairs. This is achieved by adding a shape argument to the function, which can then be used by the call to `plus_minus_generic`.

```
bell00_generic :: (QShape a qa ca) => a -> Circ (qa, qa)
bell00_generic shape = do
    qa <- plus_minus_generic shape
    (qa, qb) <- share_generic qa
    return (qa, qb)
```

The changes to Alice's function are very similar to those we have seen already.

```
alice_generic :: (QShape a qa ca) => qa -> qa -> Circ (ca,ca)
alice_generic q a = do
    (a, q) <- mapBinary controlled_not a q
    q <- mapUnary hadamard q
    (x,y) <- measure (q,a)
    return (x,y)
```

For Bob's function, we need a way of mapping classically controlled X- and Z-rotations over the input bits and qubits. The function `mapBinary_c` is similar to `mapBinary`, except that it expects a function of type `Qubit` → `Bit` → `Circ` (`Qubit`, `Bit`). Also, whereas the `controlled_not` function is a built-in operator, the classically controlled X and Z rotations are not. We use a `where` clause to define a generic `controlled_gate` function locally.

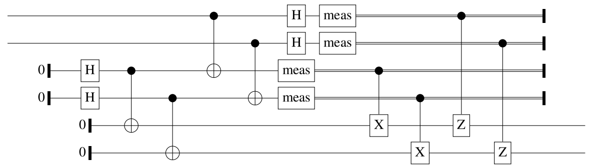
```
bob_generic :: (QShape a qa ca) => qa -> (ca,ca) -> Circ qa
bob_generic b (x,y) = do
     (b, y) <- mapBinary_c (controlled_gate gate_X) b y
     (b, x) <- mapBinary_c (controlled_gate gate_Z) b x
     cdiscard (x,y)
     return b
  where
    controlled_gate gate b x = do
      gate b `controlled` x
      return (b,x)
```

The various parts of the generic teleportation function can now be tied together.

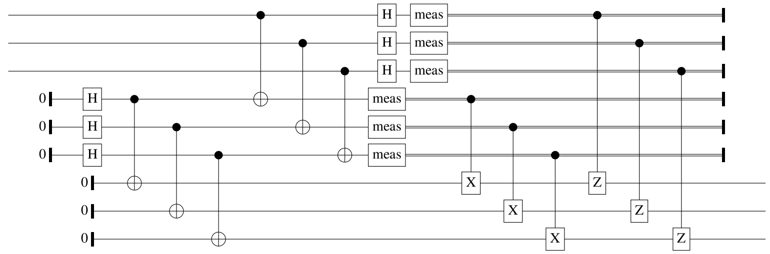
```
teleport_generic :: (QData qa) => qa -> Circ qa
teleport_generic q = do
    (a,b) <- bell00_generic (qc_false q)
    (x,y) <- alice_generic q a
    b <- bob_generic b (x,y)
    return b
```

Note that a generic Quipper function defines a *family* of circuits, one for each data type. To be able to print specific members of this family, we must replace the `print_simple` operator by the more general `print_generic`. The difference is that `print_generic` takes additional arguments to determine which instance of the circuit family to print. We show examples for teleporting a pair of qubits, and a list of three qubits:

```
print_generic PDF teleport_generic (qubit, qubit)
```

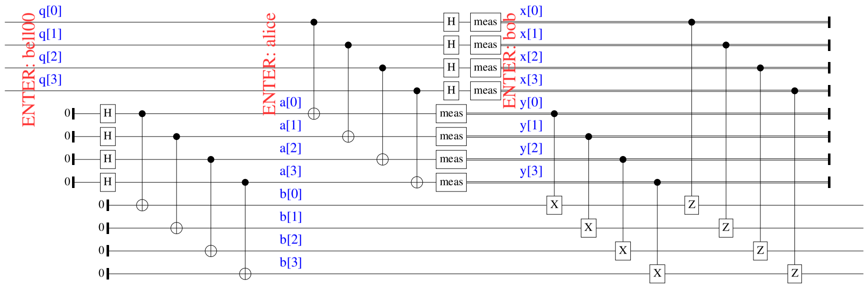



```
print_generic PDF teleport_generic [qubit,qubit,qubit]
```



Comments and labels. When reading very large circuits, it is sometimes hard to keep track of what each part of the circuit is doing, or which wires certain variables correspond to. As a convenience to the programmer, Quipper offers a way of adding comments and labels to a circuit:

```
teleport_generic_labeled :: (QData qa) => qa -> Circ qa
teleport_generic_labeled q = do
    comment_with_label "ENTER: bell00" q "q"
    (a,b) <- bell00_generic (qc_false q)
    comment_with_label "ENTER: alice" (a,b) ("a","b")
    (x,y) <- alice_generic q a
    comment_with_label "ENTER: bob" (x,y) ("x","y")
    b <- bob_generic b (x,y)
    return b
```



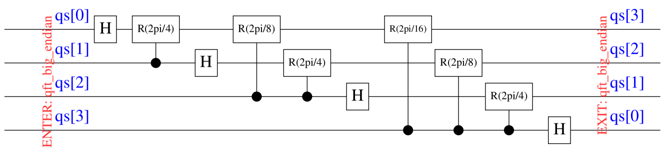
2.2 The quantum Fourier transform and quantum addition

Recursion. In Quipper it is possible to write circuit producing functions that are recursive over any parameters known at circuit generation time. Notably, we can write functions that are recursive over the shape of an input, such as a list of qubits. For example, consider the quantum Fourier transform, or QFT, which lends itself nicely to a recursive definition. The function `qft'` is defined over a list of qubits. We provide two base cases for the recursion. If the input list is empty, the circuit itself is empty. If the input is a singleton qubit, then the QFT is just the Hadamard gate. For the recursive case, the circuit for the QFT for $n+1$ qubits consists of the circuit for the n qubit QFT, followed by a set of rotations over all $n+1$ qubits. This set of rotations can also be defined in terms of a recursive function, which we call `rotations`. Also, `rGate` m is a built-in Quipper operator that represents the z-rotation by $\frac{2\pi i}{2^m}$.

```
qft' :: [Qubit] -> Circ [Qubit]
qft' [] = return []
qft' [x] = do
  hadamard x
  return [x]
qft' (x:xs) = do
  xs' <- qft' xs
  xs'' <- rotations x xs' (length xs')
  x' <- hadamard x
  return (x':xs'')
 where
  rotations :: Qubit -> [Qubit] -> Int -> Circ [Qubit]
  rotations _ [] _ = return []
  rotations c (q:qs) n = do
    qs' <- rotations c qs n
    let m = ((n + 1) - length qs)
    q' <- rGate m q `controlled` c
    return (q':qs')
```

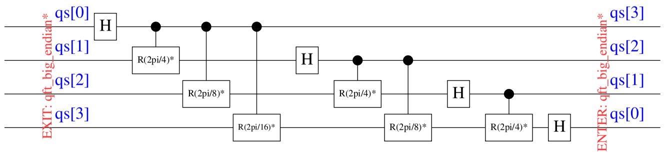
The function `qft'` expects its list of input qubits in little-endian order, but returns the output in big-endian order. Because this is confusing, we wrap it in another function `qft_big_endian`, which simply reverses the order of the input qubits. In Quipper, this is done not by swapping wires in a circuit, but by reordering *references* to wires; Quipper will attach the rest of the circuit appropriately.

```
qft_big_endian :: [Qubit] -> Circ [Qubit]
qft_big_endian qs = do
  comment_with_label "ENTER: qft_big_endian" qs "qs"
  qs <- qft' (reverse qs)
  comment_with_label "EXIT: qft_big_endian" qs "qs"
  return qs
```



Circuit-level operations. Most operators we have seen so far work at the level of gates, i.e., their effect is to append gates one by one to a circuit under construction. Quipper also has the idiom of circuit-level operations, which are operations that can be applied to circuits as a whole. One example is the printing of circuits, but there are also circuit-level operations that can be used while constructing circuits. These often take a circuit generating function as input, and produce a new circuit generating function as an output, which can then be used just like any other circuit generating function. A useful example is the operator `reverse_generic_endo`, which reverses a whole circuit. The following function computes the inverse of the QFT.

```
inverse_qft_big_endian :: [Qubit] -> Circ [Qubit]
inverse_qft_big_endian = reverse_generic_endo qft_big_endian
```

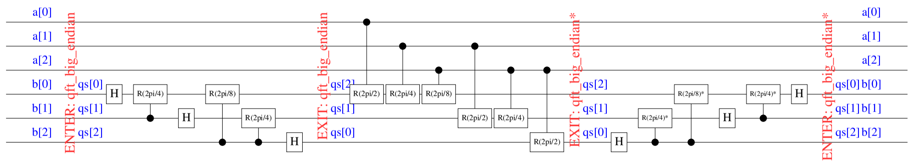


A quantum adder. As an application of the QFT, we look at a quantum circuit that performs addition [7], without the use of ancilla qubits. The circuit uses a QFT as a basis change. The inverse QFT is then applied at the end to change back to the computational basis. The part of the circuit that performs the actual addition, between the two uses of the QFT, once again lends itself to a recursive definition.

```
qft_adder :: [Qubit] -> [Qubit] -> Circ ()
qft_adder _ [] = return ()
qft_adder as (b:bs) = do
  qft_adder' as b 1
  qft_adder (tail as) bs
 where
  qft_adder' :: [Qubit] -> Qubit -> Int -> Circ [Qubit]
  qft_adder' [] _ _ = return []
  qft_adder' (a:as) b n = do
    b <- rGate n b `controlled` a
    qft_adder' as b (n+1)
```

The pattern of applying an initial computation, followed by some operation, followed by the inverse of the initial computation, is quite common in quantum computation. For this reason, Quipper provides a circuit-level operator `with_computed`, which automatically takes care of applying the inverse computation at the end. We use this here to complete the quantum addition circuit, using the QFT as the initial computation to be inverted at the end.

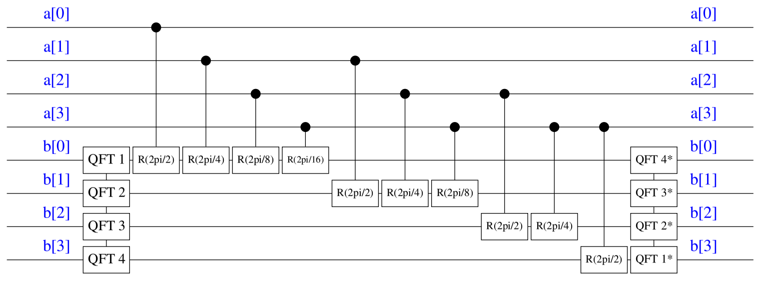
```
qft_add_in_place :: [Qubit] -> [Qubit] -> Circ ([Qubit], [Qubit])
qft_add_in_place a b = do
  label (a,b) ("a","b")
  with_computed (qft_big_endian b) $ \b' -> do
    qft_adder a (reverse b')
  label (a,b) ("a","b")
  return (a,b)
```

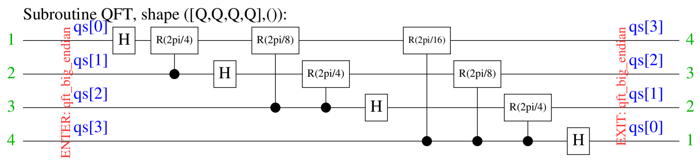


Boxed subcircuits. In many quantum algorithms, the same subcircuit is reused multiple times, which can cause a lot of duplication in circuits. Quipper helps alleviate such duplication by providing a hierarchical model of circuits, in the form of boxed subcircuits. A circuit can be *boxed*, and then reused multiple times as a subcircuit in a larger circuit. This means that the boxed subcircuit only needs to be generated once, and then a call to the boxed subcircuit is placed in the main circuit, whenever the subcircuit would appear. Quipper also permits an iteration count to be attached to a boxed subcircuit call.

A subcircuit can be boxed by using the `box` operator, which takes as its arguments a name and a function to be boxed. Here, we replicate the previous example, but with the QFT boxed.

```
qft_add_in_place_boxed :: [Qubit] -> [Qubit] -> Circ ([Qubit], [Qubit])
qft_add_in_place_boxed a b = do
  label (a,b) ("a","b")
  with_computed (box "QFT" qft_big_endian b) $ \b’ -> do
   qft_adder a (reverse b’)
  label (a,b) ("a","b")
  return (a,b)
```





Simulation of circuits. Unlike many quantum programming languages in the literature, Quipper was not designed as a front-end language for a quantum simulator; rather, it was designed to control an actual (future) quantum computer. Therefore, non-physical operations are not provided in Quipper. Nevertheless, during development and testing (and in the absence of an actual quantum computer), it is useful to be able to run simulations. Quipper provides three different simulators, which can be used depending on which gates are used within a circuit.

– Classical simulation - efficiently simulates classical circuits.
– Stabilizer simulation - efficiently simulates Clifford group circuits [1].
– Quantum simulation - simulates any circuit (with exponential overhead).

The simulators are generic: they take any circuit producing function and convert it into a function acting on the boolean counterparts to the quantum data types used in the circuit. Both the stabilizer simulator, and the quantum simulator are probabilistic.

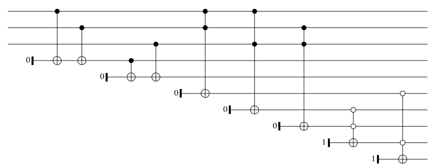
2.3 Quantum circuits from classical functions

Generating circuits from classical code. A notable feature of Quipper is the ability to automatically generate reversible circuits from ordinary functional programs. This is achieved by inserting the Quipper keyword `build_circuit` right before the classical function definition. This causes Quipper to define a new circuit generating function, with the same name as the given classical function, preceded by `template_`, where any `Bool` arguments in the type are changed to `Qubit`. We found that this language feature is useful when defining many of the oracles that appear in quantum algorithms, as they are often of a classical nature, but need to be applied to a quantum register. We have used this feature, for example, to implement a quantum library for real fixed-point arithmetic. The following example shows a single-bit full adder. A quantum function named `template_adder` will be automatically generated.

```
build_circuit
adder :: (Bool,Bool,Bool) -> (Bool,Bool)
adder (a,b,carry_in) = (s,carry_out)
 where
  s = bool_xor (bool_xor a b) carry_in
  carry_out = (a && b) || (a && carry_in) || (b && carry_in)
```

The helper function `unpack` is used to tidy up the type of any circuit produced using the `build_circuit` keyword, by removing some unnecessary occurrences of the `Circ` operator.

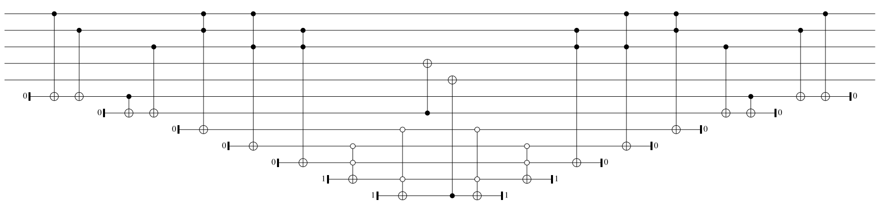
```
adder_circ :: (Qubit,Qubit,Qubit)
    -> Circ (Qubit,Qubit)
adder_circ = unpack template_adder
```



The `build_circuit` feature is implemented using a Haskell extension known as *Template Haskell*; this gives programs access to their own syntax tree in parsed form. Because of this generality, essentially arbitrary Haskell functions can be used with the `build_circuit` keyword. However, the programmer must supply quantum templates for any library functions that are used, unless they are among the standard templates already predefined by Quipper.

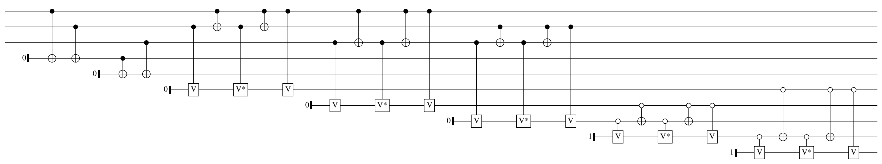
Synthesis of reversible circuits. The circuit produced by `adder_circ` is not a self-contained reversible circuit, as the automatic transformation introduces ancilla qubits that may be left in an indeterminate state, possibly entangled with the outputs. The Quipper operator `classical_to_reversible` turns a circuit $f :: a \to \texttt{Circ}\ b$ into a reversible circuit $f' :: (a, b) \to \texttt{Circ}\,(a, b)$, ensuring that any ancillas are suitably un-computed and terminated, provided that f uses only reversible primitives.

```
adder_reversible :: ((Qubit,Qubit,Qubit),(Qubit,Qubit))
   -> Circ ((Qubit,Qubit,Qubit),(Qubit,Qubit))
adder_reversible = classical_to_reversible adder_circ
```



Circuit transformations. Quipper provides a means for transforming circuits, on-the-fly, at circuit generation time. This allows for transformations such as gate decompositions, or adding certain types of error-correcting codes. Quipper provides some pre-defined transformers, as well as an extensible framework for user-defined transformers. Example transformers include the simulators, as well as a transformer to decompose circuits to only binary gates, or binary gates plus the Toffoli gate. In the following example, we apply the binary gate decomposition transformer to the adder circuit.

```
adder_circ_b :: (Qubit,Qubit,Qubit) -> Circ (Qubit,Qubit)
adder_circ_b = decompose_generic Binary adder_circ
```



3 Final remarks

3.1 Scalability and resource estimation

As we have seen, there are various things that Quipper can do with a generated circuit. However, when defining large circuits, it isn't always feasible to generate the circuit in its entirety. Quipper provides a mechanism by which one can count the resources associated with a circuit (e.g., number of gates, number of qubits, number of ancillas). Combining this feature with boxed subcircuits, we have been able to do resource estimation for some very large circuits. For example, our Quipper implementation of the triangle finding algorithm [14] produces a circuit containing over 30 trillion gates, which can be counted in under two minutes on a 1.2GHz laptop.

3.2 Prior art

There have been a number of quantum programming languages introduced in the literature (see [8]). Among the languages that have actually been implemented are Ömer's QCL [16], a C-style language optimized for quantum simulation; the Quantum IO Monad [2], which is a quantum programming language also embedded in Haskell; and Giles's LQPL [9], a functional quantum programming language with linear types. However, most of the languages that can be found in the literature are not shown to be scalable to large problem sizes.

The problem of generating circuit descriptions from functional programs has also been studied outside of the realm of quantum computing; see, e.g., [4,6].

3.3 Conclusion

Quipper has many language features, and only a selection of them have been discussed in this introductory paper. The Quipper distribution also includes some libraries of commonly used quantum functions. For example, we provide an extensive library of arithmetic functions, both for integer arithmetic and fixed-point real arithmetic; and functions for random access to a quantum register using a quantum index. Although Quipper is still in active development, we feel that the current stable release is a full-featured and scalable language. Many of the improvements that we are hoping to make are to the type system, such as introducing linear types, which will allow for more type errors to be caught at the initial compilation stage, as opposed to at circuit generation time.

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