MATH 3330: Applied Graph Theory

ASSIGNMENT #1 *Due Tue. Jan 26*

1. For the graph G=(V, E, endpts) with formal specification given by

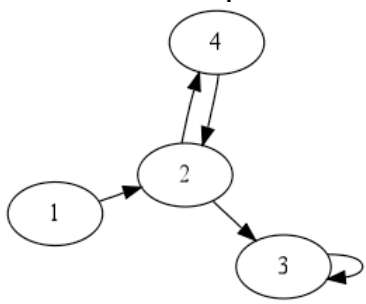   V={v,w,x,y,z}; E={a,b,c,d}; endpts(a)={v,w}; endpts(b)={w,x};
   endpts(c)={v,w}; endpts(d)={x,x},

   a) construct the incidence table for G,
   b) give the degree sequence of G, and
   c) construct a line drawing of G.

2. Construct a line drawing for the mixed graph with vertex set
   V={v,w,x,y,z} and edge set E={e,f,g,h}, and the following incidence
   table

Edges	e	f	g	h
endpts	v	y^h	w	w
	x^h	v	z	v

3. Give a formal specification for the given digraph:

4. With the methods shown in class, determine whether each of the
   following sequences is graphic:
   a) <7,6,6,5,4,3,2,1>
   b) <7,7,6,5,4,4,3,2>
   c) <5,5,4,4,2,2,1,1>
   d) <5,5,5,4,2,1,1,1>

5. Design an appropriate graph or digraph to model an ultimate Frisbee tournament involving 5 teams to be scheduled such that each team plays two other teams.

6. Consider bipartite graphs:
   a) What is the maximum possible number of edges in a simple bipartite graph on n vertices?
   b) Draw the smallest possible non-bipartite graph.
   c) Draw a 2-regular bipartite graph that is not $K_{2,2}$.

7. For each of the following graphs,

   a) P_n
   b) C_n
   c) K_n and
   d) $K_{m,n}$

   determine the size of each of the following:

   i)   the smallest dominating set,
   ii) the smallest vertex cover,
   iii) a largest independent set of vertices, and
   iv) a maximum matching.

8. For each of the following graphs,

   a) P_n for $n \geq 3$,
   b) C_n for $n \geq 4$,
   c) K_n for $n \geq 3$,
   d) $K_{m,n}$ for $m \geq n \geq 3$, and
   e) and the Petersen graph,

   determine each of the following:

   i)   the diameter,
   ii) the radius,
   iii) and central vertices of the indicated graph.