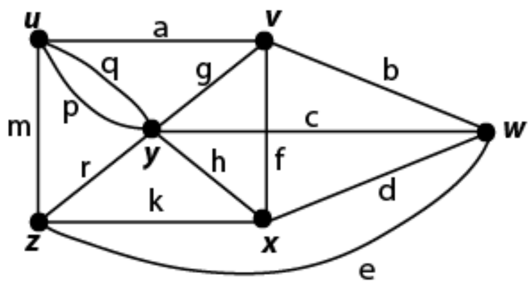
MATH 3330: Applied Graph Theory

ASSIGNMENT #3 *SOLUTIONS*

For problems 1 to 3, please use the following graph:



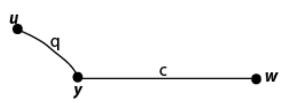
1. Determine whether the given vertex and edge subsets, W and D, respectively, form a subgraph of the graph above.

   a) W={u,w,y}; D={a,b,c}

   No: Both edges *a* and *b* require vertex *v*, which is not in W.

   b) W={u,w,y}; D={c,q}

   Yes. It is as follows:



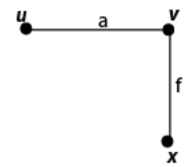
   c) W={u,w,y}; D={g,h,r}

   No: Edges *g, h* and *r* require the vertices *v, x* and *z*, respectively, as endpoints, all of which are not in W.

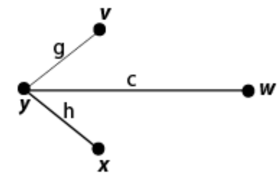
2. Find the subgraphs G(W) and G(D) induced on the given vertex and edge subsets, W and D, respectively, of the graph above.

   a) W={u,v,x}; D={c,g,h}

G(W)=

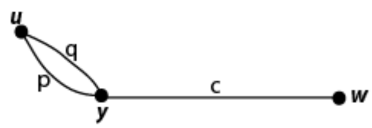


G(D)=

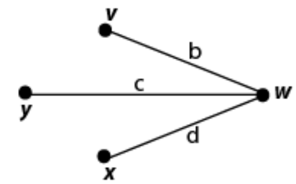


b) W={u,w,y}; D={b,c,d}

G(W)=

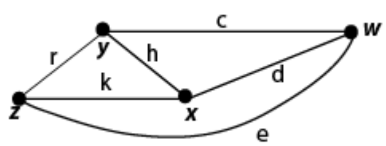


G(D)=

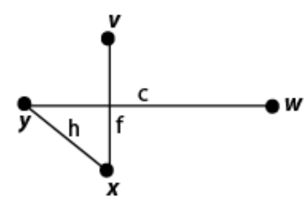


c) W={w,x,y,z}; D={c,f,h}

G(W)=



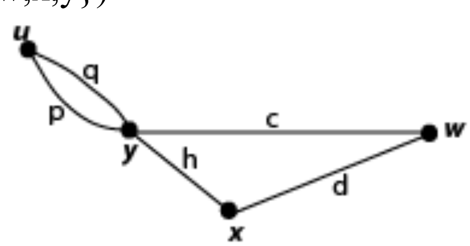
G(D)=



3. Find the local subgraphs of the given vertex in the graph above.

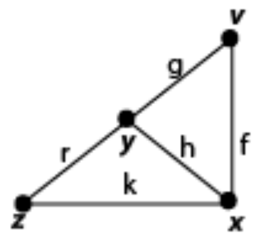
a) v

L(v)=G(N(v))=G({u,w,x,y})=



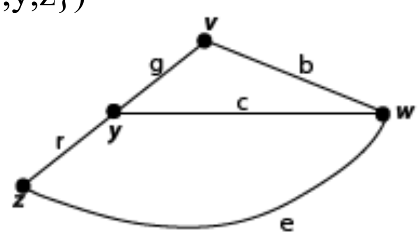
b) w

L(w)=G(N(w))=G({v,x,y,z})=

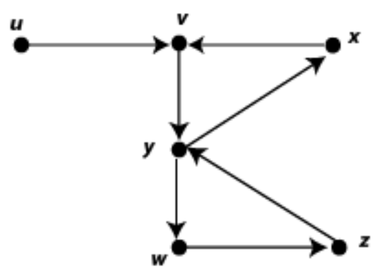


c) x

L(x)=G(N(x))=G({v,w,y,z})=

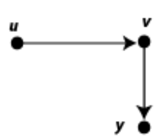


4. Find the subdigraph G(U) induced on the given vertex subset U of the digraph below.

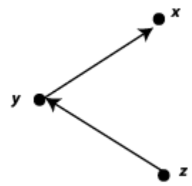


a) U={y}

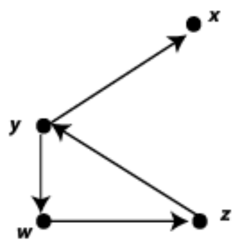
b) U={u,v,y}



c) U={x,y,z}



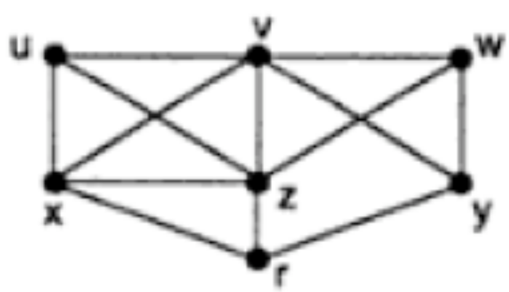
d) U={w,x,y,z}



5. For each of the following graphs,

    i) Find all of the cliques in the given graph.
    ii) Give the clique number.
    iii) Find all the maximal independent sets.
    iv) Give the independence number.
    v) Find the center.

a)



i) Cliques:
{r,x,z}, {r,y}, {u,v,x,z}, {v,w,y}, {v,w,z}

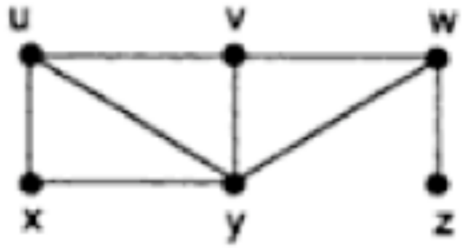
ii) Clique number = 4.

iii) Maximal independent sets:
{r,u,w}, {r,v}, {u,y}, {w,x}, {x,y}, {y,z}

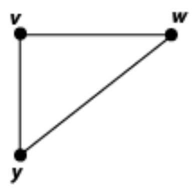
iv) Independence number=3.

v) ecc(r)= ecc(u)= ecc(v)= ecc(w)= ecc(x)= ecc(y)= ecc(z)=2
$\Rightarrow$ Center is the entire graph.

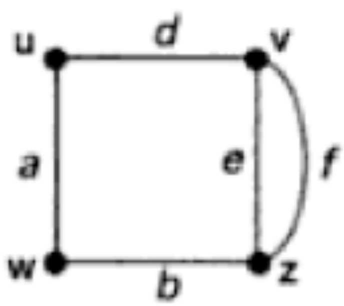
b)



i) Cliques:
{u,v,y}, {u,x,y}, {v,w,y}, {w,z}
ii) Clique number = 3.
iii) Maximal independent sets:
{u,w}, {u,z}, {v,x,z}, {w,x}, {y,z}
iv) Independence number=2.
v) ecc(u)=ecc(x)=ecc(z)=3
ecc(v)=ecc(w)=ecc(y)=2
$\Rightarrow$ Center is G({v,w,y})=



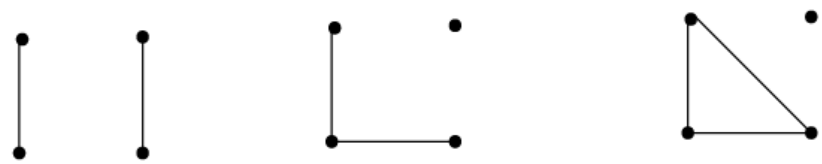
6. Find the edge-sets of all spanning trees of the graph below.



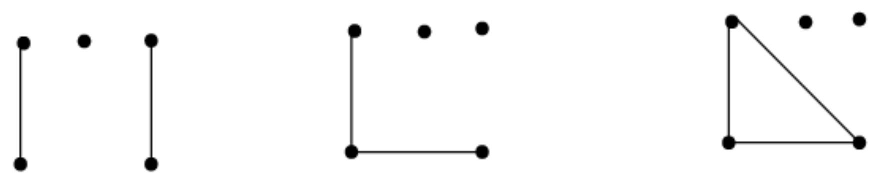
E_1={a,b,d}
E_2={a,b,e}
E_3={a,b,f}
E_4={a,d,e}
E_5={a,d,f}
E_6={b,d,e}
E_7={b,d,f}

7. Find all possible isomorphism types of the given kind of graph:

a) A simple 4-vertex graph with exactly 2 components.

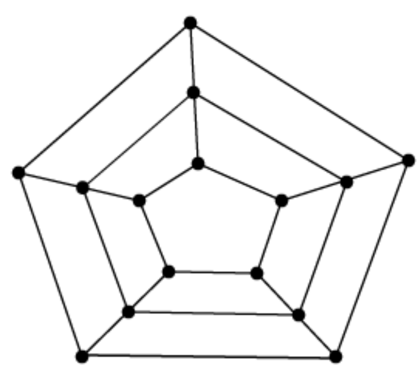


b) A simple 5-vertex graph with exactly 3 components.

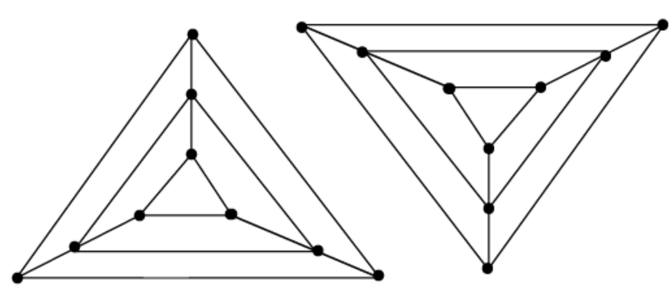


8. Draw the indicated Cartesian product:

a) $P_3 \times C_5$

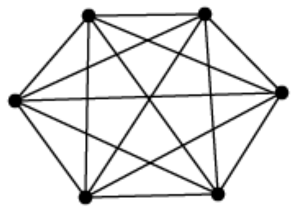


b) $P_3 \times 2K_3$

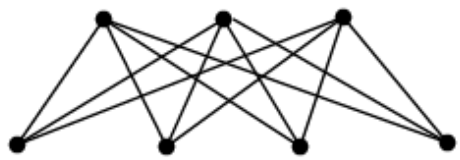


9. Draw the indicated join:

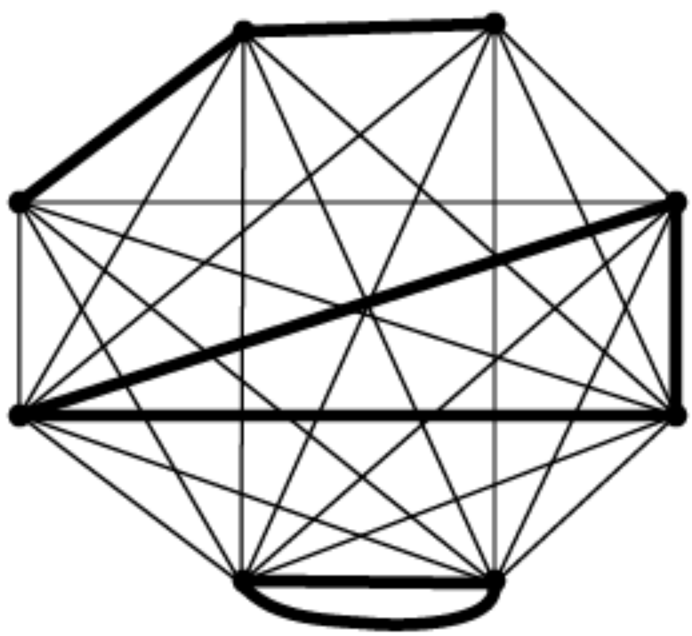
a) $K_2 + K_4 \approx K_6$



b) $3K_1 + 4K_1 \approx K_{3,4}$



c) $P_3 + K_3 + C_2$



Note that it looks suspiciously similar to a K_8, but missing one edge and then adding another multi-edge.