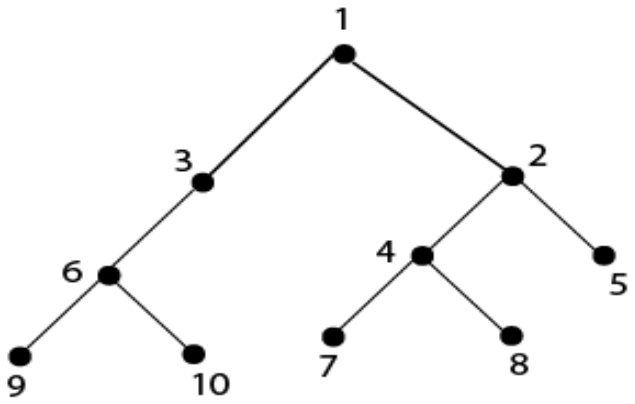
MATH 3330: Applied Graph Theory

ASSIGNMENT #5 *SOLUTIONS*

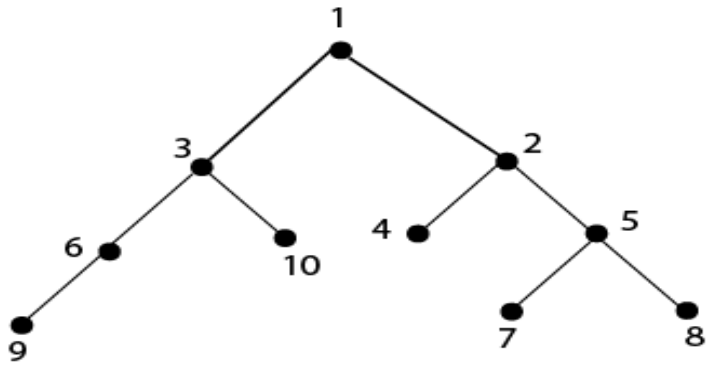
1. For the following, find the i) pre-order, ii) post-order and iii) in-order traversals of the given binary tree.

a)



i) pre-order: 1, 3, 6, 9, 10, 2, 4, 7, 8, 5
ii) post-order: 9, 10, 6, 3, 7, 8, 4, 5, 2, 1
iii) in-order: 9, 6, 10, 3, 1, 7, 4, 8, 2, 5

b)

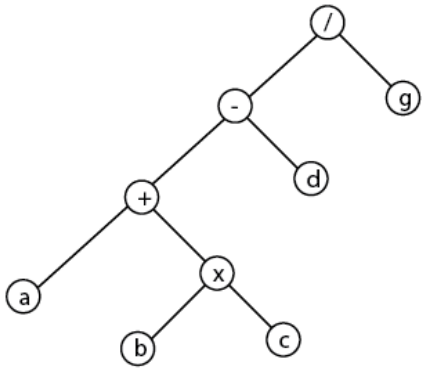


i) pre-order: 1, 3, 6, 9, 10, 2, 4, 5, 7, 8
ii) post-order: 9, 6, 10, 3, 4, 7, 8, 5, 2, 1
iii) in-order: 9, 6, 3, 10, 1, 4, 2, 7, 5, 8

2. For the following, i) represent the arithmetic expression by an expression tree, then give the ii) prefix, iii) postfix and iv) infix notations for the arithmetic expression by performing the pre-order, post-order and in-order traversals, respectively, of the expression tree.

a) $((a + (b \times c)) - d) / g$

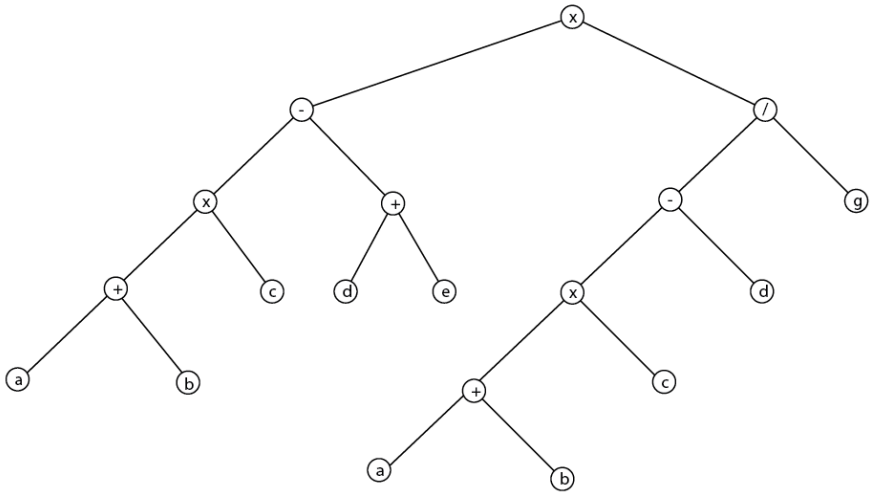
i) expression tree:



ii) prefix: /, –, +, 1, ×, b, c, d, g
iii) postfix: a, b, c, ×, +, d, –, g, /
iv) infix: a, +, b, ×, c, –, d, /, g

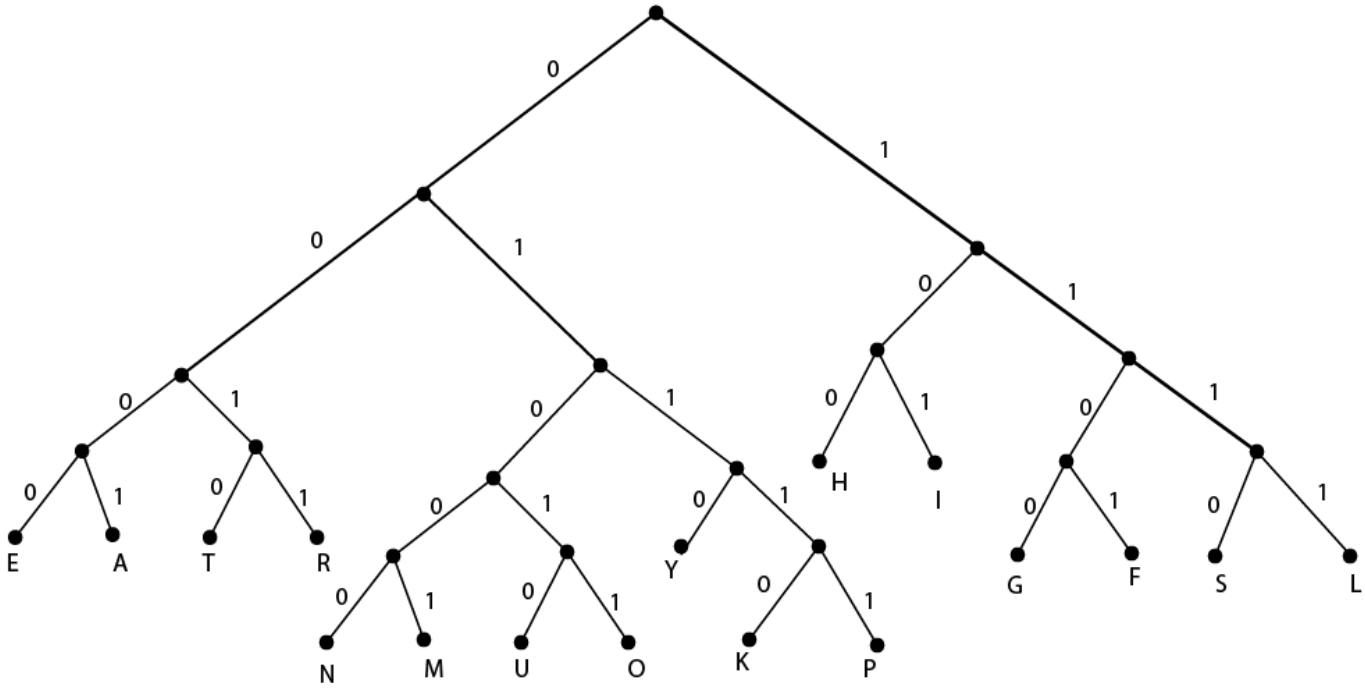
b) $(((a + b) \times c) - (d + e)) \times(((a + (b \times c)) - d) / g)$

i) expression tree:



ii) prefix: ×, –, ×, +, a, b, c, +, d, e, /, –, ×, +, a, b, c, d, g
iii) postfix: a, b, +, c, ×, d, e, +, –, a, b, +, c, ×, d, –, g, /, ×
iv) infix: a, +, b, ×, c, –, d, +, e, ×, a, +, b, ×, c, –, d, /, g

3. Use the given Huffman tree to decode the following strings



a) 101 1111101011100000 1100001100010111100
00101000000101100110110

I LIKE GRAPH THEORY
(Note: Technically, no spaces were passed)

b) 0100100010010100 1011110 1101010100 1000

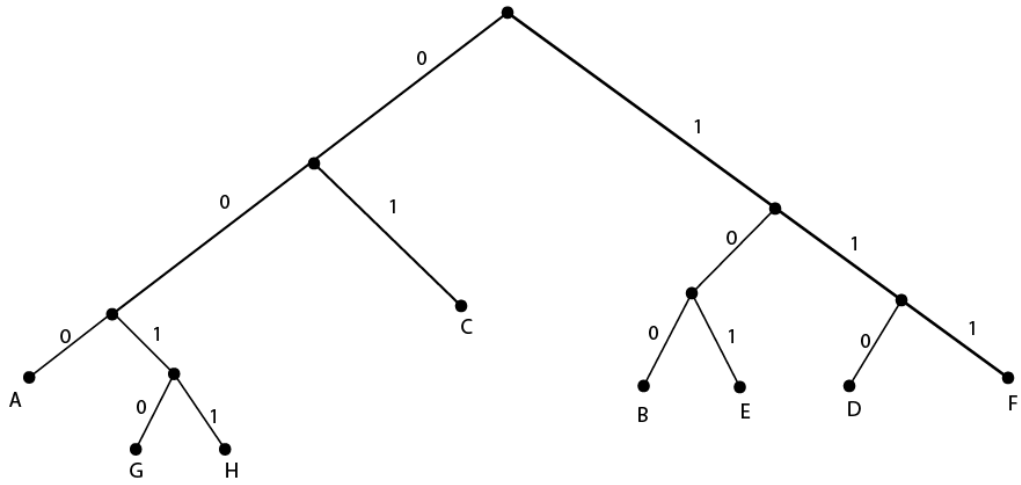
MATH IS FUN
(Again, no spaces were passed)

4. For the given list of symbols and weights,
   i) construct a Huffman code using left-to-right ordering to break ties,
   ii) calculate its average weighted length,
   iii) encode "DEFACED" and
   iv) encode "BAGGAGE".

a)

Letter	A	B	C	D	E	F	G	H
frequency	.1	.15	.2	.17	.13	.15	.05	.05

i) Huffman code:



The above Huffman code has the following encoding scheme:

Letter	A	B	C	D	E	F	G	H
Codeword	000	100	01	110	101	111	0010	0011

ii) Average weighted length
$= (3\times.1)+(3\times.15)+(2\times.2)+(3\times.17)+(3\times.13)+(3\times.15)+(4\times.05)+(4\times.05)$
$= .3 + .45 + .4 + .51 + .39 + .45 + .2 + .2$
$= 2.9$

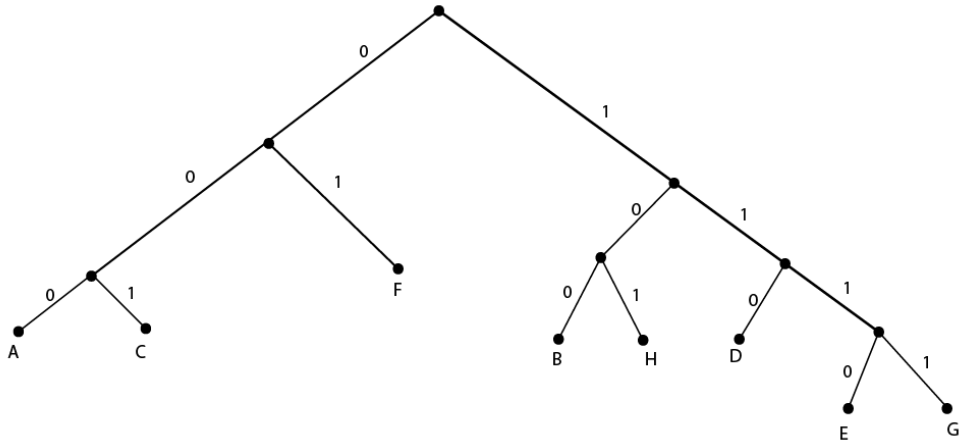
iii) "DEFACED" = 110101111000011011110

iv) "BAGGAGE" = 100000001000100000010101

b)

Letter	A	B	C	D	E	F	G	H
frequency	.15	.1	.15	.12	.08	.25	.05	.1

i) Huffman code:



The above Huffman code has the following encoding scheme:

Letter	A	B	C	D	E	F	G	H
Codeword	000	100	001	110	1110	01	1111	101

ii) Average weighted length
= (3×.15)+(3×.1)+(3×.15)+(3×.12)+(4×.08)+(2×.25)+(4×.05)+(3×.1)
= .45 + .3 + .45 + .36 +.32 +.5 +.2 +.3
= 2.88

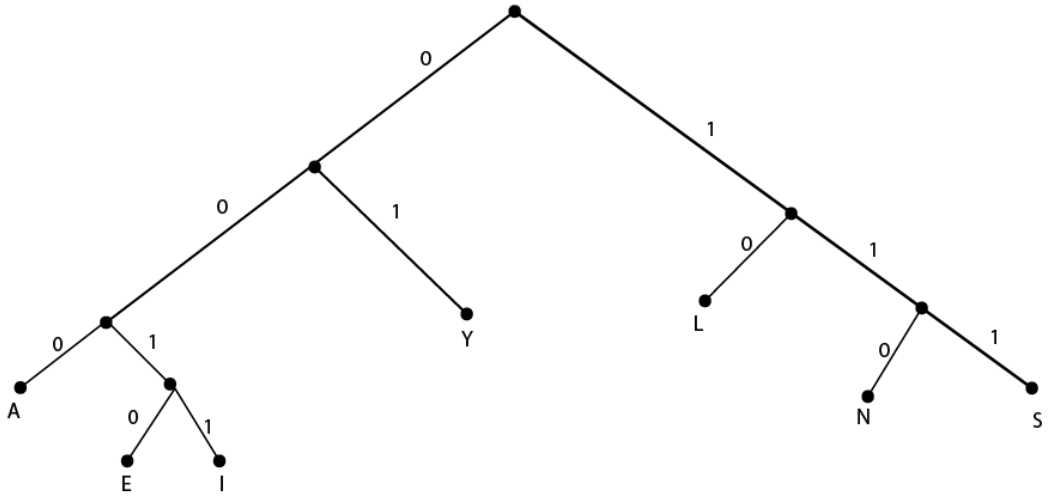
iii) "DEFACED" = 11011100100000111110110
iv) "BAGGAGE" = 10000011111111000111111110

5. Use the Huffman algorithm to construct a prefix code so that the following line is encoded using the shortest possible bitstring. Disregard the blank when encoding.

SILLY SALLY ANNE

Letter	A	E	I	L	N	S	Y
frequency	$\frac{1}{7}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Huffman code:



The above Huffman code has the following encoding scheme:

Letter	A	E	I	L	N	S	Y
Codeword	000	0010	0011	10	110	111	01

Average weighted length

$$= \left(3 \times \frac{1}{7}\right) + \left(4 \times \frac{1}{14}\right) + \left(4 \times \frac{1}{14}\right) + \left(2 \times \frac{2}{7}\right) + \left(3 \times \frac{1}{7}\right) + \left(3 \times \frac{1}{7}\right) + \left(2 \times \frac{1}{7}\right)$$

$$= \frac{3}{7} + \frac{2}{7} + \frac{2}{7} + \frac{4}{7} + \frac{3}{7} + \frac{3}{7} + \frac{2}{7}$$

$$= \frac{17}{7}$$

Encoded:

"SILLY SALLY ANNE"

= 111001110100 1 111000101001 000110110 0010