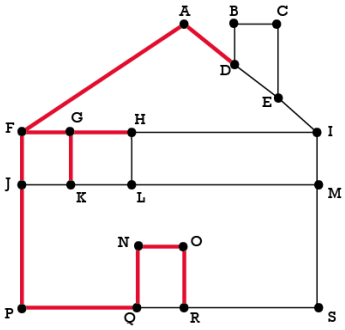
MATH 3330: Applied Graph Theory

ASSIGNMENT #6 *SOLUTIONS*

1. For the following graph and the given tree (bold edges), indicate the set of frontier edges for the given tree (bold edges). Give edges by their endpoints (i.e. XY would be edge adjacent to vertices X and Y). For each frontier edge in this set, indicate the tree vertices.

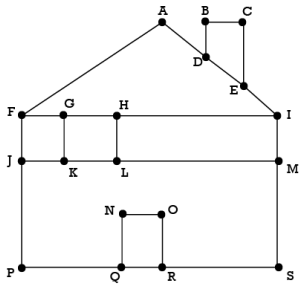


Frontier edge	DB	DE	HI	HL	KL	RS
Tree vertex	D	D	H	H	K	R

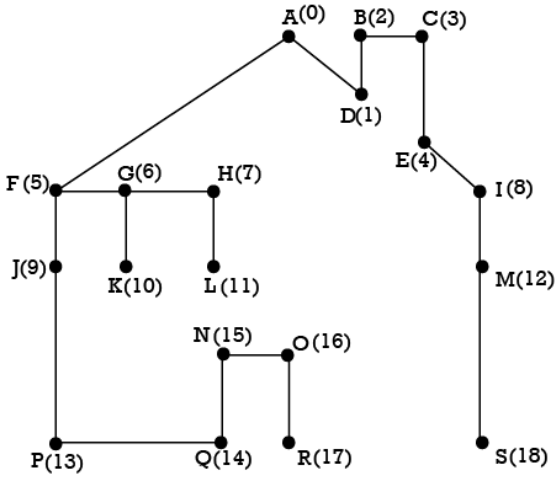
2. For which graphs are the depth-first and breadth-first trees that result identical, no matter what the default priority and starting vertex are?

   Trees.
   Reason: Consider the DFS tree. If there are any edges in the tree that are not part of the graph, then they must be "shortcuts" between a vertex and one of its ancestors. Bur if this same tree is also a BFS tree, then the path through the tree is a shortest path. So no shortcuts are possible. Therefore, there are no edges except those in the tree, and thus the graph itself is a tree.

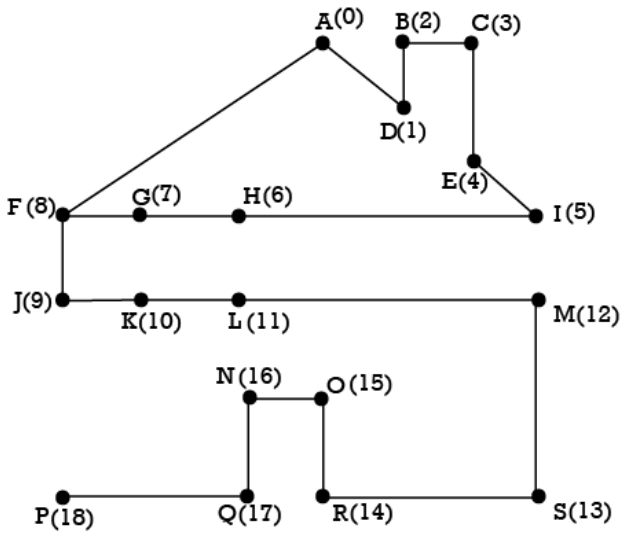
Please refer to the graph below for questions 3 through 5:



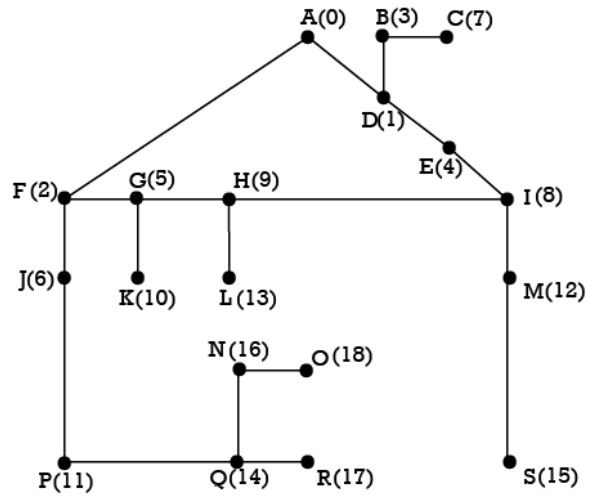
3. Draw the output tree that results when the Tree-Growing algorithm (Alg. 4.1.1) is applied to the above graph, starting at vertex A. Include discovery numbers and use lexicographic order as the default priority.



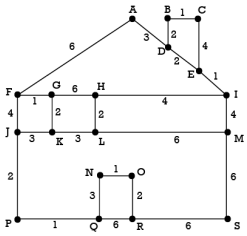
4. Draw the depth-first tree (or dfs-tree) that results when the Depth-First Search algorithm (Alg. 4.2.1) is applied to the above graph, starting at vertex A. Include dfnumbers and use lexicographic order as the default priority.



5. Draw the breadth-first tree (or bfs-tree) that results when the Breadth-First Search algorithm (Alg. 4.2.2) is applied to the above graph, starting at vertex A. Include discovery numbers and use lexicographic order as the default priority.

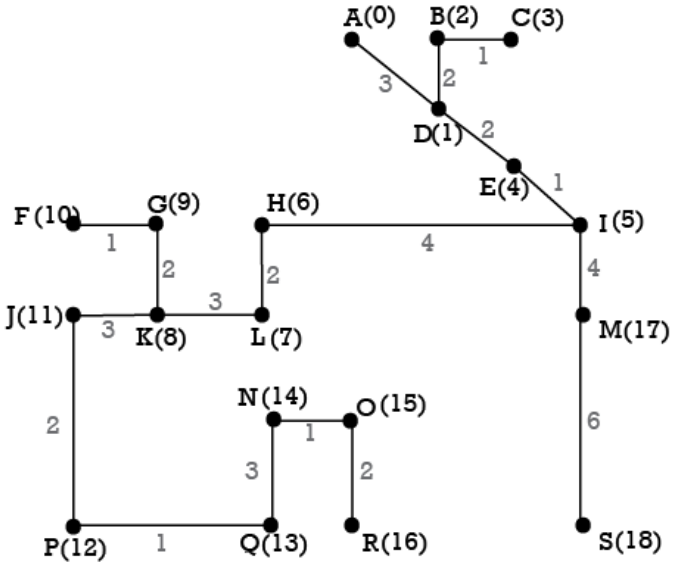


Please refer to the weighted graph below for questions 5 and 6:



6. Through application of Prim's algorithm (Alg. 4.3.1) on the above weighted graph, starting at vertex A:

   a) Draw the minimum spanning tree that results. Include discovery numbers and resolve ties using lexicographic order.

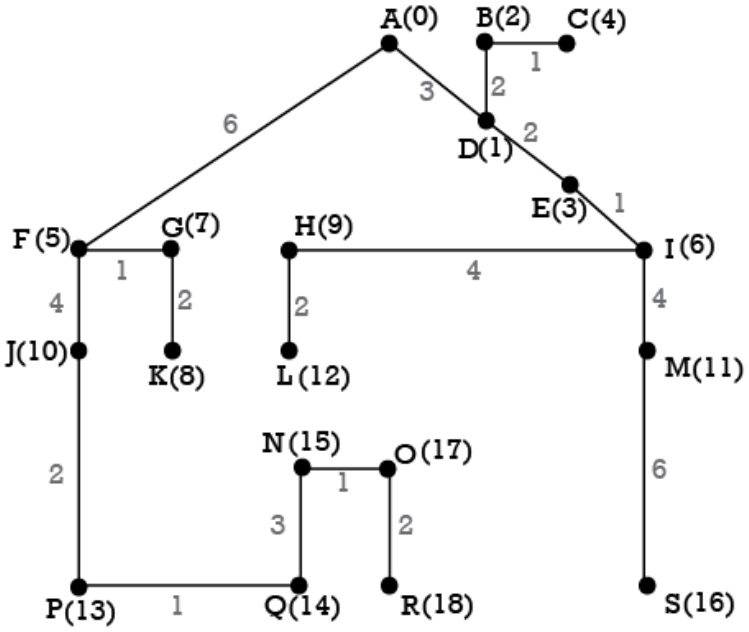


   b) Indicate the total weight of the resulting tree.

      Total weight=43

7. Apply Dijkstra's algorithm (Alg. 4.3.2) on the above weighted graph, starting at vertex A:

   a) Draw the shortest-path tree that results. Resolve ties using lexicographic order.



   b) Give a table indicating for each vertex in the graph: i) the discovery number and ii) its distance from A.

vertex	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
discovery #	0	2	4	1	3	5	7	9	6	10	8	12	11	15	17	13	14	18	16
distance from A	0	5	6	3	5	6	7	10	6	10	9	12	10	16	17	12	13	19	16