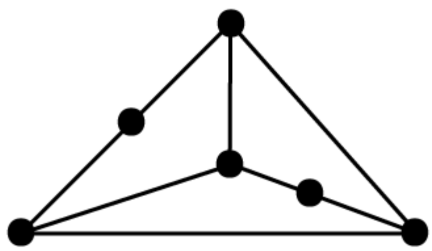
MATH 3330: Applied Graph Theory

ASSIGNMENT #8 *Due Thu. Apr. 1 by 3pm*

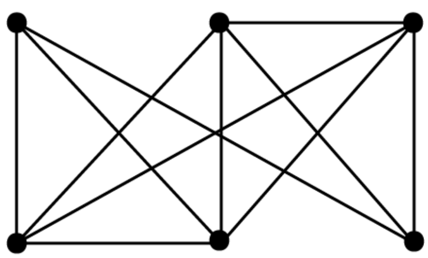
1. For each of the following, assign a minimum vertex-coloring to the graph and prove that it is a minimum coloring:

   a) K_3+C_5

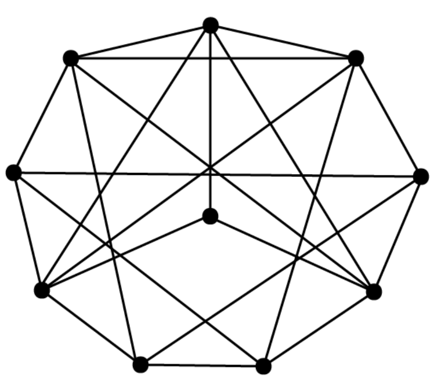
   b)



   c)



   d)



2. Find $\chi(G)$ for the Petersen graph.

3. A *k*-chromatic graph is said to be **critically *k*-chromatic** (or a *k*-critical graph) if $\chi(G - v) = k - 1$ for every vertex *v* of *G*.

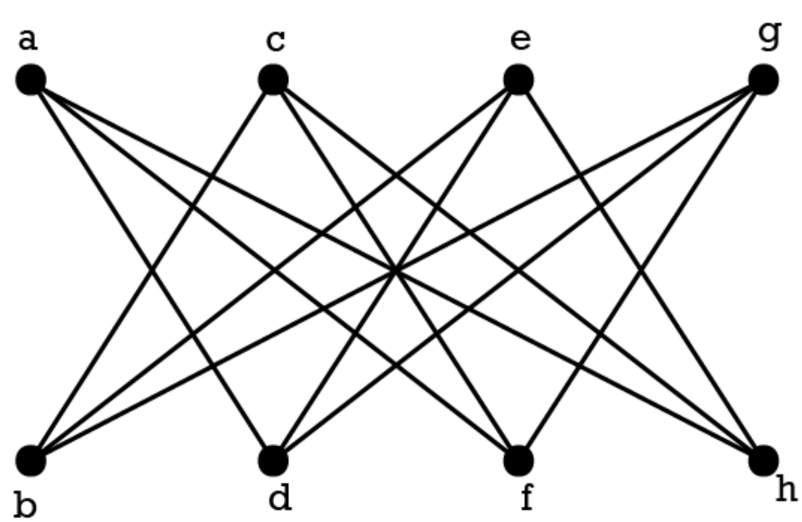
   a) Give an example of a *k*-chromatic graph that is not critically *k*-chromatic.

   b) Characterize critically 2-chromatic graphs.

   c) Characterize critically 3-chromatic graphs.

4. Prove that a k-chromatic graph G has at least k vertices of degree at least k – 1. (Hint: make use of the definition above.)

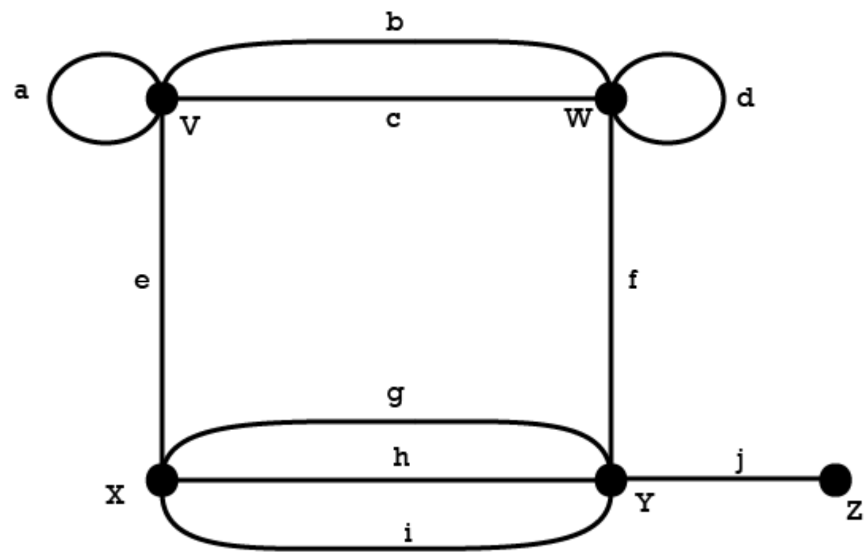
5. Apply the sequential vertex-coloring algorithm (alg. 9.1.1) to this bipartite graph, using lexicographic ordering as your vertex order.



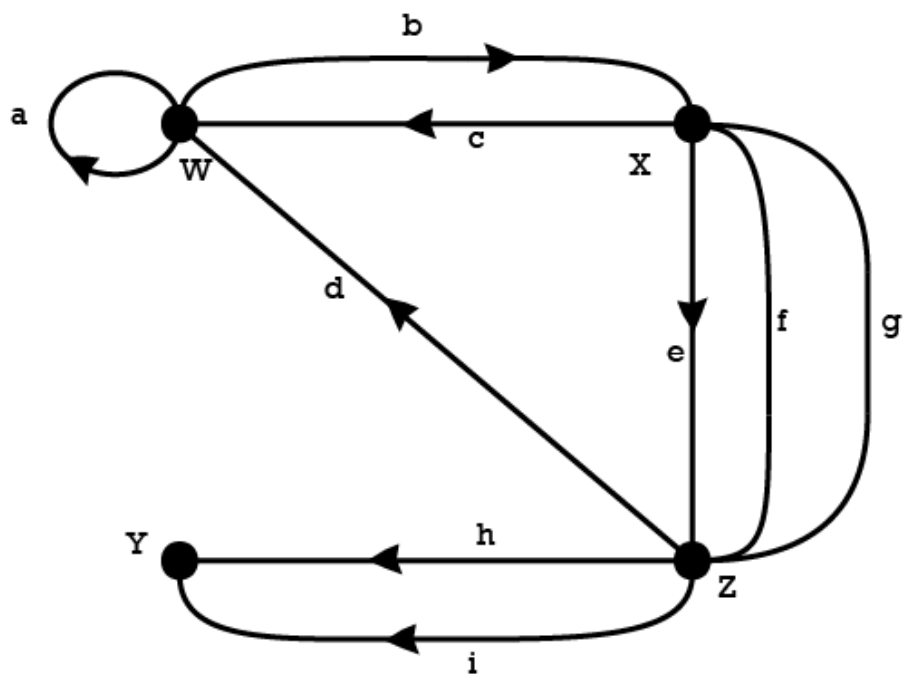
6. Prove that adding an edge to a graph increases its chromatic number by at most one.

7. For the following graph and digraph, determine the matrices A_G, I_G, and table $I_{V:E}(G)$ and, where appropriate, tables $in_{V:E}(G)$ and $out_{V:E}(G)$.

a)



b)



8. Describe the adjacency matrix of each of the following graph families:
   a) K_n
   b) P_n
   c) C_n
   d) Q_n

9. Draw the graph G that has the following adjacency matrix:

$$A_G = \begin{matrix} & a & b & c & d \\ a & 2 & 1 & 1 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 1 & 1 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{matrix}$$

10. Draw the graph G that has the following incidence matrix:

$$I_G = \begin{matrix} & a & b & c & d & e \\ w & 1 & 2 & 1 & 0 & 0 \\ x & 0 & 0 & 1 & 1 & 1 \\ y & 0 & 0 & 0 & 0 & 0 \\ z & 1 & 0 & 0 & 1 & 1 \end{matrix}$$

11. Draw the digraph D that has the following incidence matrix:

$$I_D = \begin{matrix} & a & b & c & d & e \\ w & -1 & 1 & 0 & 0 & 0 \\ x & 0 & 0 & 0 & 1 & 0 \\ y & 0 & -1 & 1 & 0 & 0 \\ z & 1 & 0 & -1 & -1 & 2 \end{matrix}$$