Craig Sutton (Dartmouth College & IAS 2023 - 2024)

Towards Hearing the Fundamental Building Blocks of Three-Dimensional Spaces

In 1966, Marc Kac asked whether it is possible to hear the shape of a drum. Specifically, he wondered about the degree to which a planar domain Ω is uniquely characterized by the eigenvalues of its associated Laplace operator (i.e., the spectrum of the domain). This question has sparked decades of ongoing research regarding the relationship between the spectrum of a Riemannian manifold and its underlying geometry. And, while there are numerous examples of geometrically distinct manifolds that have identical spectra, it is expected/hoped that certain special classes of manifolds can be characterized by their spectra.

Partially inspired by the positive resolution of the Geometrization conjecture—which illuminates the central role locally homogeneous spaces play in our understanding of the taxonomy of three-manifolds—we explore the extent to which compact locally homogeneous three-manifolds are characterized by their spectra. Examples show that the best one can hope for is that an arbitrary compact locally homogeneous three-manifold is uniquely determined up to universal Riemannian cover by its spectrum and our results provide very strong evidence that this is indeed the case.

This is joint work with Samuel Lin (Oklahoma) & Benjamin Schmidt (Michigan State).