Class #13:

15' ① Midterm I is returned. Brief discussion of common mistakes.
↳ 3b

30' ② Polynomials:

ⓐ real/imaginary parts; complex conjugation; absolute value
ⓑ zeroes of polynomials.
ⓒ fundamental thm of algebra (statement)

pause

③ Eigenvalues

ⓐ Invariant subspaces → eigenvectors
ⓑ Intuition (examples)

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② Polynomials over $\mathbb{C}$ have all roots

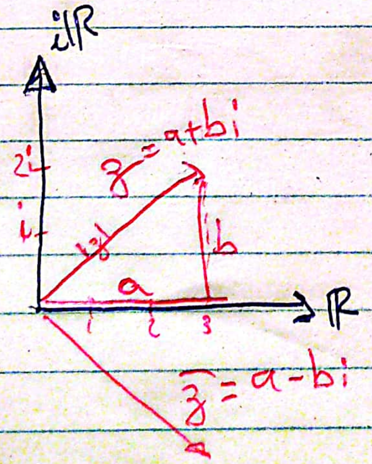
$z = \underbrace{a}_{\text{real part}} + \underbrace{b}_{\text{imaginary part}} i \in \mathbb{C}$

$\bar{z} = a - bi \in \mathbb{C}$

↳ conjugate of z

$|z| = \sqrt{a^2 + b^2} \in \mathbb{R}$

↳ length / absolute value of z



↳ behind the scenes: isomorphic (real) vector spaces $\mathbb{R}^2 \simeq \mathbb{C}$

Def A number $\lambda \in \mathbb{C}$ is a zero (= root) of $p \in \mathcal{P}(\mathbb{C})$ if $p(\lambda) = 0$

Prop If λ is a root of p, then $p(z) = (z - \lambda)q(z)$

Cor $\deg p = m \Rightarrow p$ has at most m zeroes

~~If p has~~

Thm (Fundamental thm of algebra I)
complex polynomial has a zero.

Cor (Fundamental thm of algebra II)
$p(z) = c(z - \lambda_1)(z - \lambda_2) \cdots (z - \lambda_m)$

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e.g. $p(z) = z^3 + z^2 + z^2 + 1$ has one real root ($\lambda_1 = 1$)
and two complex roots ($\lambda_2 = i$ & $\lambda_3 = -i$).
The factorization is $p(z) = (z-1)(z-i)(z+i)$

3 EIGENVALUES

~~Def~~ ~~We~~ Linear algebra is the study of linear transformations. Our goal is to understand them better. One way of doing it is by finding special bases and subspaces which play nicely w/ the transformation.

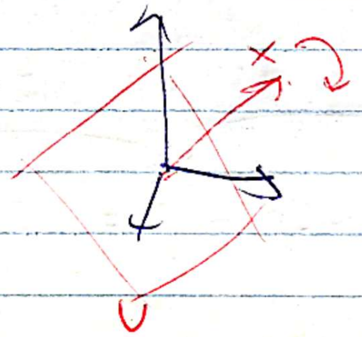
~~Notation~~ ~~OPERATOR =~~ ~~w/ the~~

Def OPERATOR = linear transformation $T: V \to V$.

Def An <u>invariant subspace</u> is a subspace $U \subseteq V$ s.t. $T(U) \subseteq U$

$T(u) \in U$.

e.g. rotation through an axis



span(x) is invariant

U is invariant

e.g. $\frac{d}{dx} : P(\mathbb{R}) \longrightarrow P(\mathbb{R})$

$P_4(\mathbb{R})$ is invariant

e.g. 0, V, null(V), range(V).

Q: invariant subspaces of dimension 1?

$U = \{ \lambda \cdot v \mid \lambda \in \mathbb{F} \} = \text{span}(v)$

$Tv \in U$? i.e. $Tv \overset{?}{=} \lambda v$ for some λ?

Def $\lambda \in \mathbb{F}$ is an eigenvalue if $\exists\, v \neq 0$ $Tv = \lambda v$.

e.g. $U = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; $v = \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix}$

$Uv = \begin{bmatrix} i-1 \\ i+1 \\ 0 \end{bmatrix} = (1+i) \begin{bmatrix} i \\ 1 \\ 0 \end{bmatrix}$

$T(x,y,z) = (7x+3z,\ 3x+6y+9z,\ -6y)$

$T(3,1,-1) = (18, 6, -6) = 6(3,1,-1)$