Seminário de Categorias da USP

sexta-feira, 17 de novembro de 2023 13:00

Timeline:

~~CIÊNCIAS MOLECULARES~~ → Math-Phys-Cat (nLab | Twitter | Youtube | Zulip), UFMG → DALHOUSIE UNIVERSITY

UFMG ↓

Faculty:

- Geoff Cruttwell (Mount Allison)
- Robert Dawson (Saint Mary's)
- Darien DeWolf (St. Francis Xavier)
- Toby Kenney (Dalhousie)
- Theo Johnson-Freyd (Dalhousie)
- Mitja Mastnak (Saint Mary's)
- Bob Paré (Dalhousie)
- Dorette Pronk (Dalhousie)
- Bob Rosebrugh (Mount Allison)
- Julien Ross (Dalhousie)
- Peter Selinger (Dalhousie)
- Richard Wood (Dalhousie)

Postdoctoral Researchers:

- Christopher Dean (Dalhousie)
- Jonathan Gallagher (Dalhousie)
- Andre Kornell (Dalhousie)
- Dongho Lee (Dalhousie)

69–71'

Lawvere

Dalhousie Gazette

Vol. 103 January 22, 1971 Number 13

Le Chateau — SCOTIA SQUARE, 5179 GOTTINGEN ST., 5520 SPRING GARDEN RD, HALIFAX, N.S.

Classroom Clamp-down

Quebec purges

On Friday, January 8, the Quebec Teachers Corporation has criticized the provincial education department for its handling of an investigation into alleged political indoctrination in classrooms.

In a statement on Friday, January 8, the 70,000 member teachers' union called the issue a "leaking political football," and said: "An administration which is not even capable of dealing with a few complaints in an organization of 100,000 teachers without resorting to publicity tricks is obviously in an alarming situation."

The day before, Education minister Guy St. Pierre was quoted as saying 50 teachers would be brought before a special committee investigating the political activity of teachers in classrooms. The setting up of this committee

Math dept. moves to remove three

by Lloyd Buchinski

An internationally known researcher and professor in the field of mathematics, Dr. F. W. Lawvere, is being dismissed from Dalhousie "in a move which rings of McCarthyism"

Tingley denied that this was the reason for the decision of the Appointments Committee.

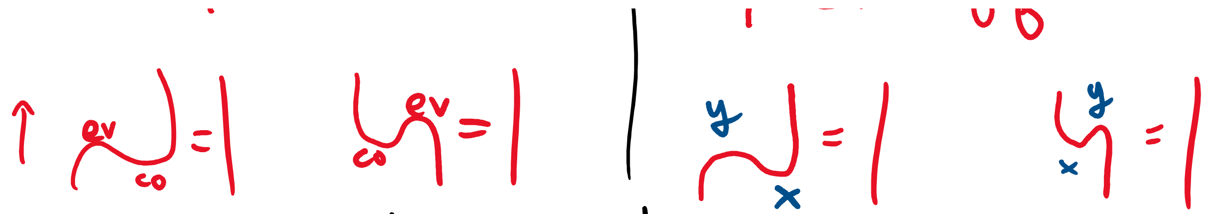
Since no real reasons, academic or otherwise, havw been given for Lawvere's dismissal, it seems to be based

vague. "The presence of Dr. Lawvere in the department causes stresses and strains to the detriment of its proper functioning," and "he has to some extent used a class for a political forum." The class

Escadas de 2-categorias → projeto de doc

① Motivação: dualizabilidade em n-categorias

dualizabilidade	"adjunctabilidade"
$(C, \otimes, 1)$ sim.	B = bicategoria
x x^R obj	mor $f: x \to y$; $f^R: y \to x$
mor $\varepsilon: x \otimes x^R \to 1$	2mor $\varepsilon: f f^R \Rightarrow 1_y$
$\eta: 1 \longrightarrow x^R \otimes x$	$\eta: 1_x \Rightarrow f^R f$

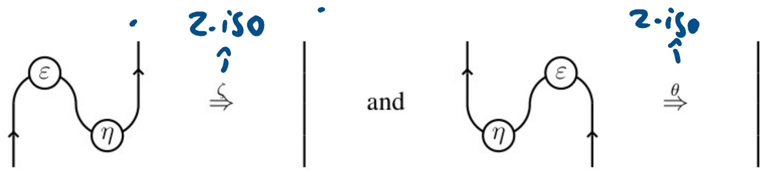


Dualizabilidade em 2-categorias:

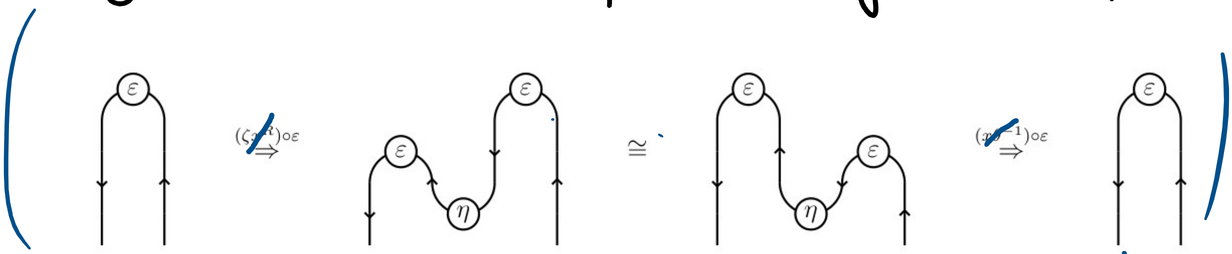
$(\mathcal{B},\otimes,1)$ = bicategoria monoidal simétrica → [CSP11]

dualizability data: $\begin{cases} \varepsilon : x^R \otimes x \longrightarrow 1 \\ \eta : 1 \longrightarrow x \otimes x^R \end{cases}$

Identidades triangulares viram data:



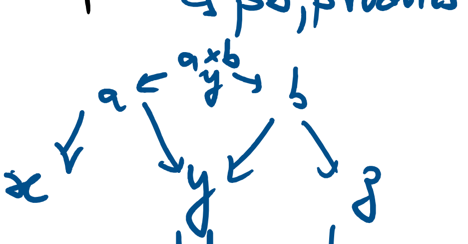
e a coerência é empurrada um grau acima:



$= L_\varepsilon$

Exemplos de bicategorias compactas: → tem duais p/ todo objeto

- Span(T) ↳ pb, produtos



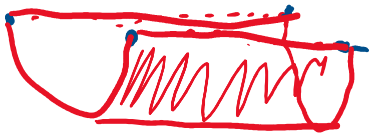
× → produto categórico

- Rel ↗ +
  - os conjuntos · $X \xrightarrow{R \subseteq X \times Y} Y$
  - ↳ 2-mor = implicação
  - ↳ monóides fracos : $M \times M \xrightarrow{R} M$
    ≅
    quantales em $\mathcal{P}(M)$

$Cob_{d,d-1,d-2}$

$Cob_{d,d-1,d-2}$

$Cob_{2,1,0}$ ↳ variedades de dimensão $d, d-1, d-2$



- TQFT: $Cob_{d,d-1} \xrightarrow{F} Vect_{\mathbb{C}}$
- TQFT 1-estendida: $Cob_{d,d-1,d-2} \xrightarrow{F} 2Vect_{\mathbb{C}}$ ↳ 2-vector spaces

Surpresa: a condição de coerência não é necessária.

↳ pra falar de duais

$\begin{cases} \varepsilon : 1 \to x \otimes x^R \\ \eta : x^R x \to 1 \end{cases}$

- Basta analizar a dualizabilidade em $(h_1(B), \otimes, 1)$ → $\begin{cases} ob \\ mor/iso \end{cases}$
- Isso é um fato sobre adjunctabilidade em 3-categorias.

$(B, \otimes, 1) \to \Omega B = C \in 3Cat$ ↳adj → $\boxed{D_e C}$

2-dualizabilidade = dualizabilidade + adjunctabilidade

$(B, \otimes, 1)$ = bicategoria simétrica

1-mor em B $\begin{cases} \varepsilon : 1 \to x x^R \\ \eta : x^R x \to 1 \end{cases}$ dual — se sim, x é 2-dualizável

↳ $\exists ?\ \varepsilon^L \dashv \varepsilon \dashv \varepsilon^R$?

$\eta^L \dashv \eta \dashv \eta^R$

2-dualizabilidade é respondida por duas bicategorias: responde questões de duas bicategorias: responde questões de existência dos duais

. $(h_1 B, \otimes, 1)$ → responde existência dos duais

. B (sem a estrutura monoidal) → questões de adjunção

Em 3-categorias: (2-adjunctabilidade)

. $h_2 C$ → ob, 1mor, 2mor/3iso

. Mor C → {1mor, 2mor, 3mor}

$f \in C \to f \dashv f^R$?

$\{\varepsilon: 1 \to f f^R,\ \eta: f^R f \to 1\} \to \{A: f \to \varepsilon \varepsilon^R,\ B: \varepsilon^R \varepsilon \to 1\}$ $.\varepsilon^R$

<u>Motivação:</u> hipótese do cobordismo

TQFT 2-estendida $\mathrm{Cob}_{d,d-1,d-2} \xrightarrow{\text{2-functor monoidal}} \{2\mathrm{Vec}_{\mathbb{C}},\ (B, \otimes, 1)\}$

TQFT d-estendida $\mathrm{Cob}_{d,d-1,\dots,1,0} \xrightarrow{\text{d-functor}} \{d\mathrm{Vec}_{\mathbb{C}},\ (C, \otimes, 1)\}$ → ???

$F(*) \in d\mathrm{Vec}_{\mathbb{C}}$

$\boxed{\text{Teorema}}$ 1. é d-dualizável

2. classifica F → "Os objetos d-dualizáveis" classificam TQFTs d-estendidas.

Em n-categorias: C = n-categoria

$h_2 C,\ h_2^1 C,\ \dots,\ h_2^{n-3} C,\ h_2^{n-2} C$

n mor, n-1 mor, $\overset{2}{n}$ mor

[n-1 mor], n-2 mor, n-3 mor

[3 mor]

2 [2 mor] ⇝ 2 mor

→ $\mathrm{Mor}(h_2 C) = h_2(h_2^1 C)$

2 [2mor] ⇝ 2mor

1 mor mor

0 ob

$\longrightarrow \mathrm{Mor}(h_2 C) = h_1(h_2 C)$

$\boxed{\text{Def}}$ Escada de ~~2-~~bi-categorias

$$B_0, B_1, \dots, B_{n-1}, B_n$$

tq $\mathrm{Mor}(B_k) \simeq h_1(B_{k+1})$

$\boxed{\text{Slogan}}$ Escadas respondem todas questões sobre dualizabilidade em n-categorias

↳ 1. dar um contexto categórico
2. descobrir o que além do dualizabilidade

② Um ambiente para escadas

Nesta apresentação eu vou falar sobre o caso estrito. Algumas variações estão no PDF.

n-categoria = categoria enriquecida sobre (n-1) cat

$\mathrm{Cat}_n = ((n-1)\mathrm{Cat})\text{-}\mathrm{Cat}$

$h_1 : \mathrm{Cat}_n \to \mathrm{Cat}$ = ob + classes de 1-mor

$h_2 : \mathrm{Cat}_n \to \mathrm{Cat}_2$ = ob + 1mor + classes de 2-mor

$\mathrm{Mor}: \mathrm{Cat}_n \to \mathrm{Cat}_{n-1} =$ $(n-1)$cat de $1, \dots, n$ morfismos

$C \mapsto \bigsqcup C(x,y)$

$h_2^{(K)} : \mathrm{Cat}_n \to \mathrm{Cat}_2 =$ $C \mapsto$ $[k-2]$ mor, $k-1$ mor, K mor

$$\mathrm{Cat}_5 \xrightarrow{h_2} \mathrm{Cat}_2 \text{ - - - - - - - - - - - - } h_2^{(0)} \quad 0,\ 1,\ [2]$$

$$\xrightarrow{\mathrm{Mor}} \mathrm{Cat}_3 \xrightarrow{h_2} \mathrm{Cat}_2 \text{ - - - - - - - - } h_2^{(1)} \quad 1,\ 2,\ [3]$$

$$\xrightarrow{\mathrm{Mor}} \mathrm{Cat}_3 \xrightarrow{h_2} \mathrm{Cat}_2 \text{ - - - - } h_2^{(2)} \quad 2,\ 3,\ [4]$$

$$\xrightarrow{\mathrm{Mor}} \mathrm{Cat}_2 \text{ - - - - } h_2^{(3)} \quad 3,\ 4,\ 5.$$

Definition 3.1. A **ladder of 2-categories** is a (possibly infinite) sequence of 2-categories $\mathcal{B} = (\mathcal{B}_0, \dots, \mathcal{B}_n)$ such that $h_1\mathcal{B}_{k+1} = \mathrm{Mor}(\mathcal{B}_k)$ for $0 \le k \le n-1$.

Ex (B_0, B_1) tq $h_1 B_1 = \mathrm{Mor}(B_0)$

$$\begin{tikzcd}
\{g(z)=f(y)\} \subseteq Z\times Y \arrow[r] \arrow[d] & Y \arrow[d, "f"] \\
Z \arrow[r, "g"] & X
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathrm{Lad}_2 \subseteq 2\mathrm{Cat}\times 2\mathrm{Cat} \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner"] & 2\mathrm{Cat} \arrow[d, "\mathrm{Mor}"] \\
2\mathrm{Cat} \arrow[r, "h_1"] & \mathrm{Cat}
\end{tikzcd}$$

~~Set~~ ~~Cat~~

Def

Categoria de escadas

$\mathrm{Lad}_n \cdot$ pb

$$
\begin{tikzcd}[column sep=small]
\mathrm{Cat}_2 \arrow[dr, "\mathrm{Mor}"'] & & \mathrm{Cat}_2 \arrow[dl, "h_1"] \arrow[dr, "\mathrm{Mor}"'] & & \mathrm{Cat}_2 \arrow[dl, "h_1"] \arrow[dr, "\mathrm{Mor}"'] & & \mathrm{Cat}_2 \arrow[dl, "h_1"] \\
& \mathrm{Cat} & & \cdots & & \mathrm{Cat} &
\end{tikzcd}
$$

Obs:

$$
\begin{tikzcd}
\mathrm{Cat}_3 \arrow[r, "\mathrm{Mor}"] \arrow[d, "h_2"'] \arrow[dr, phantom, "\wr"] & \mathrm{Cat}_2 \arrow[d, "h_1"] \\
\mathrm{Cat}_2 \arrow[r, "\mathrm{Mor}"'] & \mathrm{Cat}
\end{tikzcd}
\quad \Rightarrow \quad
\begin{tikzcd}
\mathrm{Cat}_3 \arrow[dr, dashed, "\exists!\, Ł"] \arrow[drr, bend left, "\mathrm{Mor}"] \arrow[ddr, bend right, "h_2"'] & & \\
& \mathrm{Lad}_2 \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & \mathrm{Cat}_2 \arrow[d, "h_1"] \\
& \mathrm{Cat}_2 \arrow[r, "\mathrm{Mor}"'] & \mathrm{Cat}
\end{tikzcd}
$$

$$
\begin{tikzcd}
(\bullet \to \bullet) \arrow[r] \arrow[d] & C \arrow[d] \\
\mathrm{Adj} \arrow[r] \arrow[ur, dotted] & \bullet
\end{tikzcd}
$$

adjunção caminhante

$\downarrow Ł$

$$
\begin{tikzcd}
Ł(\bullet \to \bullet) \arrow[r] \arrow[d] & Ł(C) \arrow[d] \\
Ł(\mathrm{Adj}) \arrow[r] \arrow[ur, dotted] & Ł(\bullet)
\end{tikzcd}
$$

Prop| Lad_n é acessível

Lema| Lad_n é um pullback fraco
$\longleftrightarrow$ Mor é uma isofibração

Thm MP 891| Acc é fechada por pullbacks fracos
$\subset \mathrm{Cat}$

Mor/h_1 são acessíveis $\Rightarrow$ ☒

Cor| Ł é acessível
h_2 é acessível $\Rightarrow$ ☒

③ Outras direções e futuro

- Checar fatos conhecidos sobre n-categorias
- Escadas de bicategorias

- Escadas de bicategorias
- Escadas de 2-categorias "gaunt"

↳ som 2-iso n̄ triviais

1-iso n̄ triviais

ser eq = ser igual

- dualizabilidade ✓
- mais comportada

- Escadas de $(\infty, 2)$-categorias

- teoria seja bem comportada

- Estudar outras propriedades de Lad_n

além de dualizabilidade