Topology and Cosmology

Daniel Teixeira

Dalhousie University

March 8, 2023

Cosmology and Topology

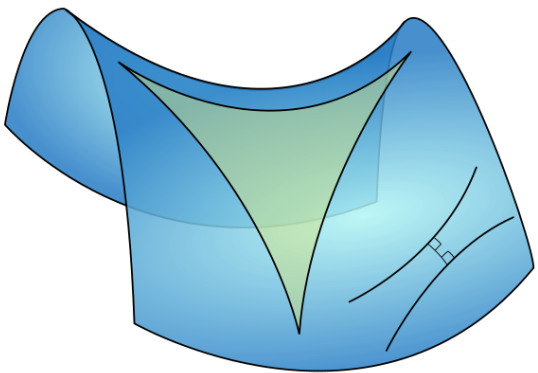
1. Our universe
2. Non-trivial spaces
3. Detecting topology

Prelude: topology

- Topology studies the **global** propeties of spaces, such as connectivity and number of holes.

Prelude: geometry

- Geometry studies the **local** structure of spaces, such as curvature, distances anda angles.



Our universe

Spacetime is a four-dimensional manifold equipped with **a Lorentzian metric** and a **time orientation** satisfying **Einstein equations**.

Our universe

Spacetime is a **four-dimensional manifold** equipped with **a Lorentzian metric** and a **time orientation** satisfying **Einstein equations**.

$\rightsquigarrow$ topology is not set by GR!

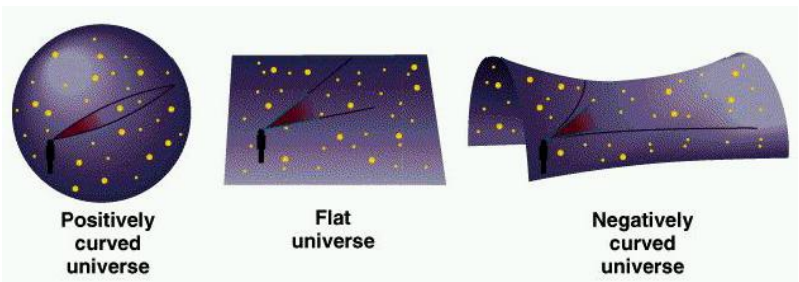
Our universe

There are physical constraints...

- Lorentzian
- Time orientability
- Orientability
- Global hyperbolicity

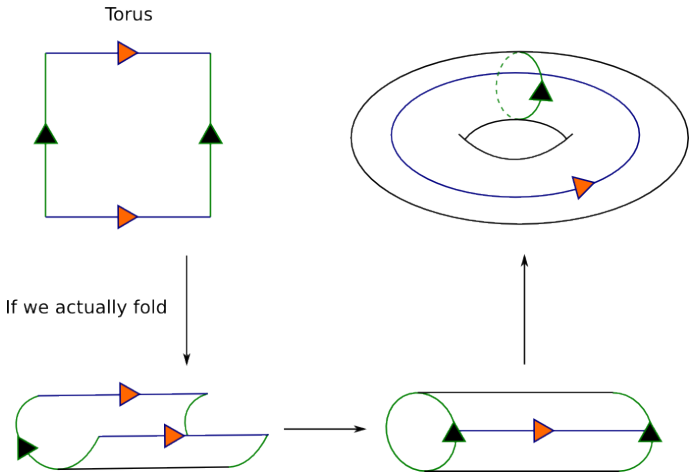
Cosmology

Cosmological principle:
the spatial slices are homogeneous and isotropic.



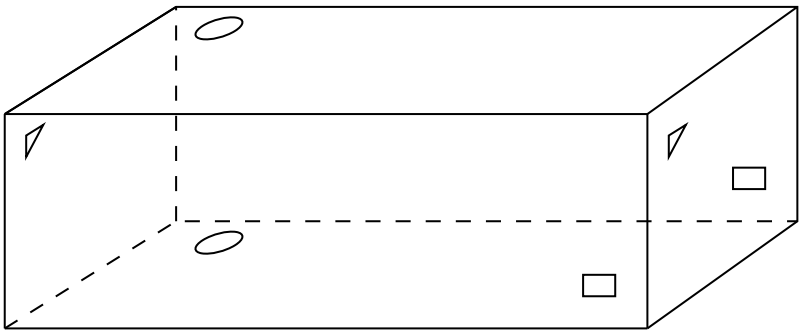
Which 3-manifolds are homogeneous and isotropic ?

A 2d interlude: the torus



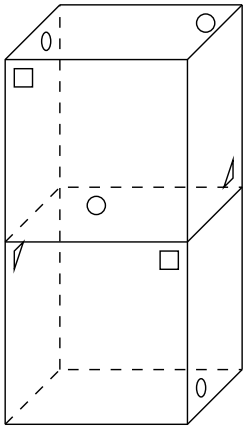
Source: https://pi.math.cornell.edu/ mec/Winter2009/Victor/part1.htm

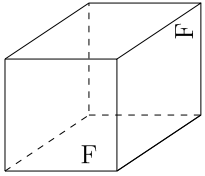
Euclidean 3-manifolds



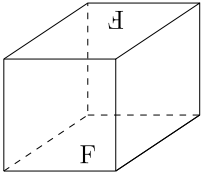
Source: https://mphitchman.com/geometry/section8-1.html

Euclidean 3-manifolds

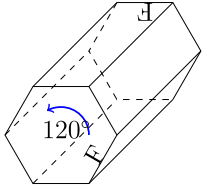




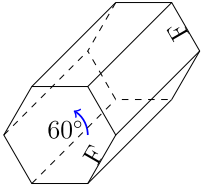
(a)



(b)



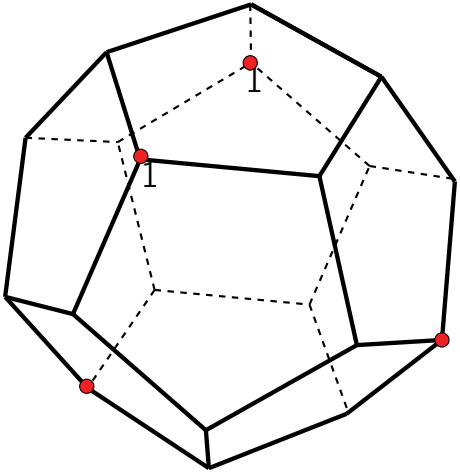
(c)



(d)

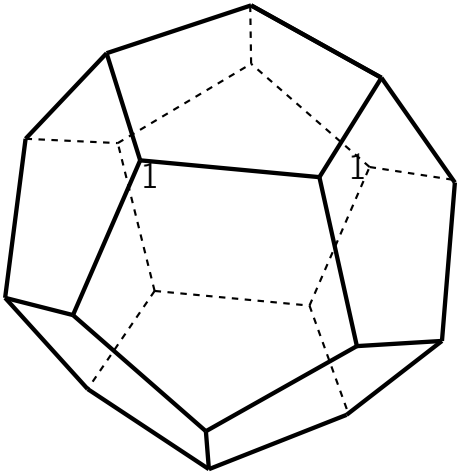
Source: https://mphitchman.com/geometry/section8-1.html

An elliptic example



Source: https://mphitchman.com/geometry/section8-1.html

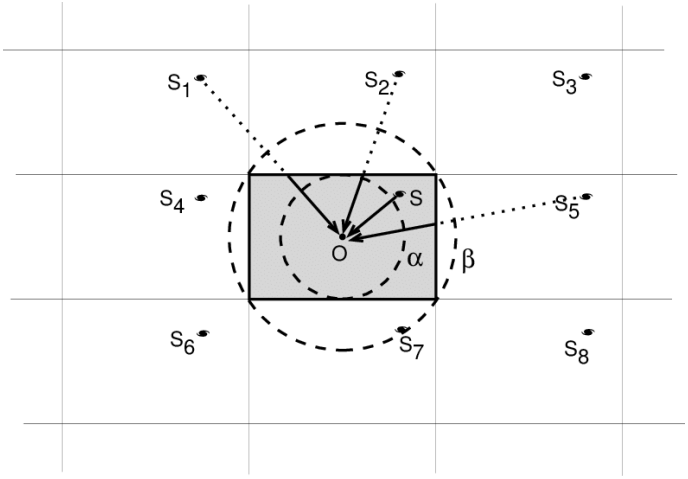
An hyperbolic example



Source: https://mphitchman.com/geometry/section8-1.html

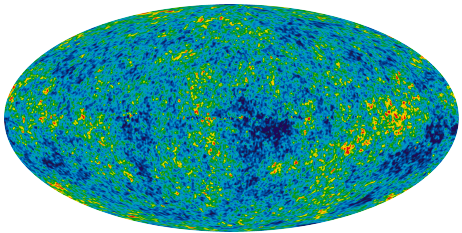
How to actually *measure* topology?

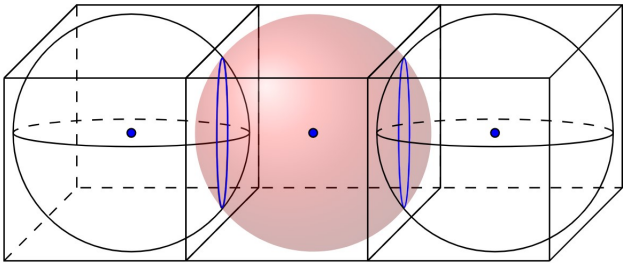
Multiple images



Source: https://mphitchman.com/geometry/section8-1.html

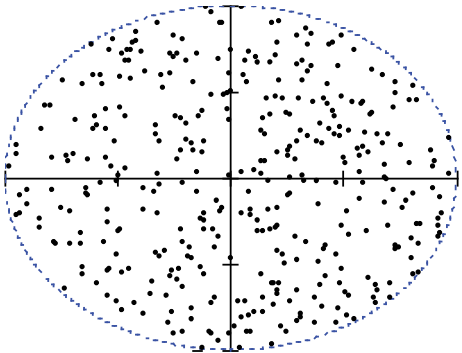
Circles in the sky

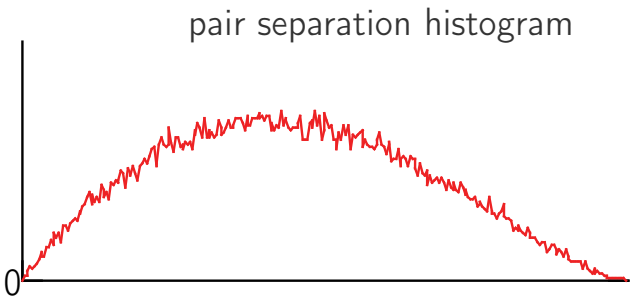




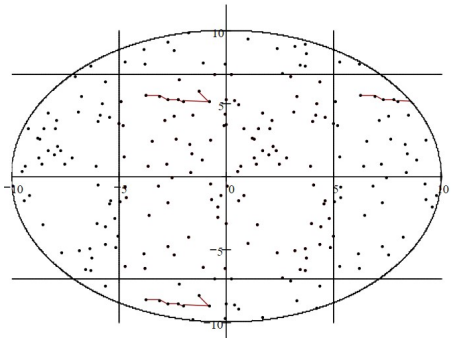
see https://arxiv.org/abs/gr-qc/9602039

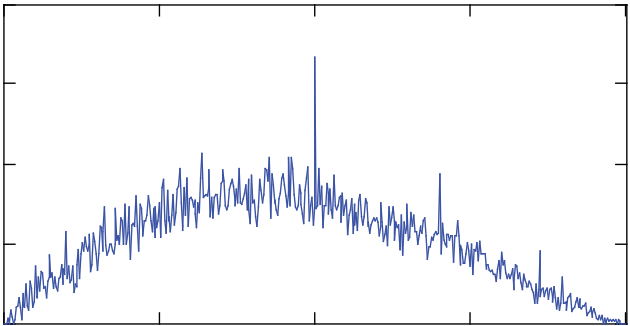
Cosmic crystallography





Cosmic crystallography





see https://arxiv.org/abs/gr-qc/9602039

Back to real life

- Planck 2013 data $\rightsquigarrow$ surveyed for toroidal + two topologies - nothing.
- Cosmic crystallography $\rightsquigarrow$ analysis of quasar datasets.
- Recent efforts
  $\rightsquigarrow$ explore different topologies in the databases
  $\rightsquigarrow$ explore the eigenmodes of the Laplacian in different topologies and see how these affect the distributions in the CMB

References

- Michael P. Hitchman, Geometry with an Introduction to Cosmic Topology. Available at: https://mphitchman.com/geometry/gct-toc.html.
- Jeffrey Weeks, Topology and geometry software. Available at: https://www.geometrygames.org/
- The games Asteroid and the special stages of Sonic the Hedgehog 3.