MATH 1115, Mathematics for Commerce
WINTER 2011
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Homework Sheet 5
Model Solutions

Each multiple choice question is worth one mark, other questions are worth two marks.

1. The number of 3-element subsets of a set with 8 elements is given by

   (A) $\frac{8\times7\times6}{5!}$

   (B) $8 \times 7 \times 6$

   (C) $\boldsymbol{\frac{8\times7\times6}{3!}}$

   (D) $\frac{8\times7\times6}{3}$

   (E) It depends on which 8-element set.

2. If we toss a fair coin 50 times. The probability that we get exactly 22 heads is:

   (A) $\frac{1}{51}$

   (B) $\frac{1}{2^{50}}$

   (C) $\boldsymbol{\frac{{}_{50}C_{22}}{2^{50}}}$

   (D) $\frac{22}{50}$

   (E) $\boldsymbol{\frac{50\times\cdots\times29}{22!\times2^{50}}}$

3. *Assuming the answers to each multiple-choice question on this homework (there are 6 multiple-choice questions) are independant, and each of the 5 answers A to E is equally likely for each question, what is the probability that each of the 5 possible answers occurs at least once?*

   The number of possible sequences of correct answers is 5^6 (by the multiplication principle). If there is at least one of every letter, then 1 letter must occur twice and the other 4 must occur once each. There are 5 possible choices for which letter occurs twice. Once this has been chosen, we have to count the number of arrangements of 6 letters, one of which is repeated. The number of possibilities is $\frac{6!}{2!} = \frac{6!}{2}$.

   (A) $\boldsymbol{\frac{5\times6!}{2\times5^6}}$

   (B) 1

   (C) $1 - \frac{4^6}{5^6}$

   (D) $\frac{6!}{2\times5^6}$

   (E) $\frac{5\times5!}{5^6}$

4. *If 4 cards are drawn at random (without replacement) from a standard deck of 52 cards, the probability that no two are the same suit is: [A standard deck contains 4 suits each of which has 13 cards.]*

   There are ${}_{52}C_4$ possible sets of 4 cards. If no two are the same suit, then there is exactly one card of each suit. To describe a set of 4 cards with one of each suit, we just need to give the rank of each card, so there are 13^4 possibilities.

   (A) $\left(\frac{3}{4}\right)^6$

   (B) $\frac{4!}{4^4}$

   (C) $\frac{13^4}{52\times51\times50\times49}$

   (D) $\boldsymbol{\frac{4!\times13^4}{52\times51\times50\times49}}$

   (E) $\frac{(13!)^4}{52!}$

5. *A company has 3 warehouses. One of them can hold 3,000 items, one of them can hold 2,500, and one of them can hold 4,500 items. The company wants to store 10,000 items in these warehouses. The number of ways in which the company can divide the items between the three warehouses is:*

   There are ${}_{10000}C_{2500}$ ways to choose which items to put in the first warehouse, and once this is chosen, there are ${}_{7500}C_{3000}$ ways to choose which of the remaining items to put in the second warehouse.

   (A) $2500 \times 3000 \times 4500$

   (B) 3^{10000}

   (C) $\frac{4500!}{2500!\times3000!}$

   (D) ${}_{10000}C_{2500} \times_{7500} C_{3000}$

   (E) ${}_{10000}C_3$

6. *If in the previous question, the company discovers there is a problem with three of the items stored in the warehouses, what is the probability that all three are in different warehouses?*

   There are ${}_{10000}C_3$ possible choices of 3 defective items. There are $2500 \times 3000 \times 4500$ choices of one item from each of the three warehouses.

   (A) $2500 \times 3000 \times 4500$ divided by the answer to the previous question.

   (B) $\boldsymbol{\frac{2500\times3000\times4500}{{}_{10000}C_3}}$

   (C) $1 - \frac{2500\times2500}{10000^3} - \frac{3000\times3000}{10000^3} - \frac{4500\times4500}{10000^3}$

   (D) $\frac{3!}{3^3}$

   (E) $\frac{{}_{2500}C_3+{}_{3000}C_3+{}_{4500}C_3}{{}_{10000}C_3}$

7. *A bank makes loans to three customers. They estimate that the first customer has probability 0.01 of defaulting (not paying the money back), the*

second has probability 0.02, and the third has probability 0.05. Assuming that these events are independant:

(a) What is the probability that all three loans are paid back?

The probability the first is paid back is $1 - 0.01 = 0.99$. The probability the second is paid back is $1 - 0.02 = 0.98$. The probability the third is paid back is $1 - 0.05 = 0.95$. Therefore the probability that all three are paid back is therefore $0.99 \times 0.98 \times 0.95$. [This is equal to 0.92169]

(b) What is the probability that at least two of the loans are paid back?

There are 4 mutually exclusive ways that at least two of the loans are paid back:

- the first two customers pay back but the third doesn't.
- the first and third customers pay back but the second doesn't.
- the last two customers pay back but the first doesn't.
- all three customers pay back.

The probabilities of these events are $0.99 \times 0.98 \times 0.05$, $0.99 \times 0.02 \times 0.95$, $0.01 \times 0.98 \times 0.95$, and $0.99 \times 0.98 \times 0.95$, respectively. The total probability is therefore $0.99 \times 0.98 \times 0.05 + 0.99 \times 0.02 \times 0.95 + 0.01 \times 0.98 \times 0.95 + 0.99 \times 0.98 \times 0.95$.

[This is equal to $0.04851 + 0.01881 + 0.00931 + 0.92169 = 0.99832$]

8. *An insurance company sets it's premium for car insurance at \$500. They estimate that the probability of a customer making a claim is $\frac{1}{200}$, in which case they will pay out \$50,000. They sell policies to 100 customers. After costs, the premiums are enough for them to pay out up to 8 claims.*

   (a) For a given set of 8 customers, what is the probability that those 8 customers make claims, and the other customers do not (assume that the claims of different customers are independant)?

   The probability that those 8 customers make claims is $\left(\frac{1}{200}\right)^8$. The probability that the other 92 customers do not make claims is $\left(\frac{199}{200}\right)^{92}$. The probability that those 8 make claims and the other 92 do not is therefore $\left(\frac{1}{200}\right)^8 \left(\frac{199}{200}\right)^{92}$.

   (b) What is the probability that the insurance company will have to pay out exactly 8 claims?

   For any set C of 8 customers, the probability that the company has to pay claims by exactly the customers in C, and not the customers in C^c is $\left(\frac{1}{200}\right)^8 \left(\frac{199}{200}\right)^{92}$. For any set C of 8 customers, let E_C be the event that the customers in the set C all make claims and the other customers do not. For two distinct sets C and D, the events E_C and E_D are mutually exclusive.

The total number of these events is ${}_{100}C_8$, so the total probability that one of these events occurs is

$$ {}_{100}C_8 \left(\frac{1}{200}\right)^8 \left(\frac{199}{200}\right)^{92} $$