MATH 2112/CSCI 2112, Discrete Structures I
Winter 2007
Toby Kenney
Homework Sheet 4
Due in: Wednesday 7th February, 1:30 PM

Compulsory questions

1 Show that for all integers n, n^2 is congruent to 0, 1, 2, or 4 modulo 7.

2 Show that $2^{13} + 3^{241}$ is divisible by 5.

3 If $7|n^3$ for some integer n, show that $7|n$.

4 Show that for any integer n, one of $n^2 + 4$ and $n^3 + n$ is divisible by 5.

5 Define the repeat of a positive integer as the number obtained by writing it twice in a row (in decimal). For example, the repeat of 364 is 364364. Find a positive integer n such that the repeat of n is equal to m^2 for some integer m.

[Hint: the repeat of n is always a multiple of n. You may find the following divisibility test useful: a number is divisible by 11 if and only if the sum of its odd digits minus the sum of its even digits is divisible by 11. For example, 1254 is divisible by 11 since the sum of its odd digits is 1+5=6 and the sum of its even digits is 2+4=6, so their difference is 0, which is divisible by 11. You won't find the answer by trial and error.]

6 Show that $3^{258} + 17$ is not the square of any integer n.

7 Solve for x – i.e. find all values of x less than the number modulo which we are working (so, for example, in (a), give all values of $x \in \{0, 1, \ldots, 12\}$) satisfying the equation:

(a) $4x \equiv 7 \pmod{13}$

(b) $8x \equiv 9 \pmod{12}$

(c) $7x \equiv 19 \pmod{24}$

(d) $12x \equiv 44 \pmod{64}$

8 Consider the collection of integers whose last digit (in decimal) is 1. The product of any two such integers is another such integer. Any such integer can therefore be factored as a product of integers of this type that cannot be written as non-trivial products of other integers of this type. For example, $7,211 = 11 \times 21 \times 31$, and 11, 21, and 31 cannot be expressed as products of integers whose last digit is 1.

Can every integer of this type be written in a unique way as such a product? Give a proof or a counterexample.

9 Are there any solutions to $x^6+y^6+3=z^6$, where x, y, and z are integers? [Hint: any solution x, y, z would also be a solution to $x^6+y^6+3 \equiv z^6 \pmod{n}$ for any integer n.]