# MATH 2113/CSCI 2113, Discrete Structures II
## Winter 2008
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### Homework Sheet 9
### Due: Wednesday 2nd April: 1:30 PM

## Compulsory questions

1 (a) Show that if we 3-colour the complete graph on 17 vertices, we get a monochromatic triangle.

(b) Is it possible to partition the set $\{1, 2, \ldots, 16\}$ into 3 sets such that no set contains any number which is the sum of two numbers in the same set (possibly the same number twice)? [Hint: Given such a partition into the sets $A$, $B$, and $C$, take a complete graph on 17 vertices labelled $v_1, v_2, \ldots, v_{17}$, and colour the edge $v_iv_j$ red if the difference $|i-j|$ is in $A$, blue if $|i-j|$ is in $B$, and green if $|i-j|$ is in $C$. What does a monochromatic triangle mean for this colouring?]

2 Suppose we colour each edge of the complete graph on 11 vertices red with probability $\frac{1}{3}$ and blue with probability $\frac{2}{3}$ (so it is always coloured either red or blue).

(a) What is the expected number of red $K_4$s?

(b) What is the expected number of blue $K_6$s?

(c) Deduce that there is a 2-colouring of the complete graph on 11 vertices without a red $K_4$ or a blue $K_6$. [Hint: $2^{14} < 3^9$, $\binom{11}{4} = 330$, $\binom{11}{6} = 396$, $3^6 = 729$.]

3 (a) Show that if we 2-colour (red and blue) the edges of the complete graph on 10 vertices, we get either a red triangle or a blue complete graph on 4 vertices.

## Bonus Question

(b) Show that if we 2-colour (red and blue) the edges of the complete graph on 9 vertices, we get either a red triangle or a blue complete graph on 4 vertices. [Hint: In part (a), you probably found a condition on a vertex that would force the existence of either a red triangle or a blue $K_4$. With 10 vertices, this condition must hold for every vertex. With 9 vertices, it doesn't need to hold for every vertex, but suppose it doesn't hold for any vertex, and consider the subgraph consisting of just the blue edges.]