

MATH 2600/STAT 2600, Theory of Interest

FALL 2010

Toby Kenney

Homework Sheet 3

Due: Thursday 14th October: 1:00 PM

1. Ms. Jones invests \$500 a month in her pension plan, which pays interest at $j_{12} = 6\%$. She makes her first investment on her 21st birthday. How much money does she have in the plan when she retires on her 65th birthday (assuming that she still makes the payment on that birthday)?
2. A car dealership allows a customer to pay for a car with monthly payments of \$200 for 5 years. If the company is charging 10% interest on these payments, what is the price of the car?
3. Jack wants to start a pension plan for his retirement. He decides that after he retires, he will need \$1,000 a month from his pension. His pension fund pays interest at $j_{12} = 5\%$.
 - (a) If he plans to have enough in his pension fund to last for 25 years, how much will he need at the time he retires (assuming the payments start one month after his retirement).
 - (b) He is currently 43. If he plans to retire at 65, i.e. exactly 22 years from today, how much does he need to contribute to the pension plan every month to achieve the target in part (a), assuming he makes his first payment today, and his last one in 22 years.
 - (c) He decides that he can only afford to put in \$200 a month. He wants his pension fund to last until he is 90 years old — i.e. exactly 47 years from today, and he still wants to be able to withdraw \$1000 a month when he starts to withdraw from his pension. When can he retire and still achieve this?
4. A man takes out a loan for \$10,000 at an interest rate $j_4 = 10\%$. He agrees to repay it with a set of quarterly payments of \$500.
 - (a) How many payments does he need to make?
 - (b) If he agrees to pay off the debt by making a balloon payment on the last payment, calculate the value of the last payment?
 - (c) If instead, he agrees to make a drop payment, what will be the value of his last payment?
5. A woman takes out a mortgage for \$100,000, to buy a house. The bank tells her that she can pay it off with monthly payments of \$1,000 for 20 years.
 - (a) What nominal rate of interest (compounded monthly) are they charging her?

(b) Suppose that she can invest her money at $j_{12} = 7\%$. Suppose further, that the house price rises with inflation of 0.5% every month. How long would she have to save up the same \$1,000 a month payments before she could buy the house with a single payment?

(c) Suppose she were given the option of renting the same house while saving up the money to buy it. What monthly rent should she be willing to pay?

6. Mr. MacDonald makes a donation of \$1,000,000 to his old University. He requests that it be used to set up a scholarship fund which pays \$20,000 a year to each of 3 students, and should last for 100 years. If the money is invested at 5% interest per year, how long does the university have to wait before it can make the first payment from the scholarship fund. [Use the theoretical method for compound interest for partial years.]
7. Dr. Proctor sets up a savings account on 1st January 2001 to pay for his childrens' education. He pays \$100 into the account each month, starting on 1st January 2001, until 1st June 2004, when he increases the payments to \$150 a month. Then from 1st April 2009 onwards, he reduces the payments back to \$100 each month. The interest rate for the account is determined each year by market conditions, and the interest rates are as given in the following table:

| Year | Interest rate |
|------|---------------|
| 2001 | 5% |
| 2002 | 5% |
| 2003 | 4% |
| 2004 | 4% |
| 2005 | 5.5% |
| 2006 | 5.5% |
| 2007 | 8% |
| 2008 | 4% |
| 2009 | 4% |
| 2010 | 5% |
| 2011 | 5% |
| 2012 | 5% |

How much is in the account when he makes his last payment on 1st August 2012?