MATH 2600/STAT 2600, Theory of Interest
FALL 2010
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Homework Sheet 7
Due: Thursday 2nd December

1. A machine has a current cost of \$40,000. The machine has an expected lifetime of 20 years. It cannot be resold when it is finished with. The maintainance costs are \$700 in the first year, and increase by \$500 in each subsequent year. The cost of capital is $j_1 = 6\%$.

   (a) What is the total capitalised cost if the machine is replaced every 20 years?

   (b) It turns out to be best to replace the machine every 15 years. What is the total capitalised cost in this case?

   (c) Another type of machine for the same task has a current cost of \$80,000, but it's price is expected to fall by 4% every year, as the technology improves. It's maintainance costs are \$2,000 a year. It also lasts 20 years. Would this machine be cheaper in the long run? [Retraining costs prevent buying the cheaper machine first, and then changing to the other machine when it becomes cheaper.]

2. A computer is bought for \$1500. It is expected to last for 3 years, after which it will have a value of \$300. Prepare a depreciation schedule using

   (a) The straight-line method.

   (b) The constant percentage method.

3. A mining company buys a mine which they estimate containes 5,000 tonnes of ore, for \$1,000,000. After the mining is finished, they expect that they will be able to sell the land for a net price (after restoration costs) of \$200,000. In the first 3 years, the company mines 1,500 tonnes of ore. What is the book value of the mine after 3 years?

4. Two standard fair dice are rolled.

   (a) What is the probability that the larger number is 4?

   (b) What is the expected value of the larger number?

5. Mr. Davis is about to retire. The total value of his pension plan is \$150,000, and it is invested at $j_1 = 6\%$. The probability of his dying this year is 1%, the probability of dying in any subsequent year is such that the overall probability of dying within n years is $\frac{n}{50}$ (i.e. the probability that he dies in year n is $\frac{1}{50-n}$).

   (a) What is the expected value of the remaining length of his life?

   (b) If he decides to withdraw \$6,000 at the start of every year until he dies, what is the expected present value of all the withdrawls?

(c) If he were to make exactly 25 withdrawls, what would the present value be? Why is this answer different from the answer in (b)?

(d) (i) How much should he withdraw every year, so that the expected present value is equal to $150,000?

(ii) Would it be a good idea for him to withdraw this amount every year?

(iii) Would it be reasonable for a large company to agree to pay him this amount (less its commission) every year until he dies, in exchange for the money in his pension plan?