MATH 2600/STAT 2600, Theory of Interest
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Midterm examination. Model Solutions

1. *An investor has a promissory note for \$3,000 in 150 days at 5% simple interest. After 40 days, she sells it to a bank which discounts notes at 4.8%*

   (a) How much does the bank pay for the note?

   The legal due date is in 153 days, at which point the value of the note is $3000(1 + 0.05 \times \frac{153}{365}) = \$3,062.88$. The bank discounts it at 4.8% for 113 days, so they pay $\frac{3062.88}{1+0.048\times\frac{113}{365}} = \$3,018.03$.

   (b) What rate of return does the original investor achieve?

   The investors rate of return is $\frac{18.03}{3000\times\frac{40}{365}} = 5.48\%$.

2. *Which of the following rates of interest is best for the investor?*

   (i) 7% compounded monthly

   (ii) 7.1% compounded quarterly

   (iii) 7.25% compounded annually

   The increase after 6 months in the 3 cases are:

   (i) $(1 + \frac{0.07}{12})^6 - 1 = 3.55\%$

   (ii) $(1 + \frac{0.071}{4})^4 - 1 = 3.58\%$

   (iii) $\frac{7.25}{2} = 3.625\%$

   So (iii) is best for the investor.

3. *A man takes out a loan for \$40,000 at $j_{12} = 3\%$. He plans to repay it with 3 equal payments: one after 6 months, one after 1 year, and the final payment after 3 years. What will these payments be?*

   Let X be the value of the payments, then an equation of value with focal date at the present time gives:

   $$40000 = X(1.0025)^{-6} + X(1.0025)^{-12} + X(1.0025)^{-36}$$

   so we get

   $$X = \frac{40000}{1.0025^{-6} + 1.0025^{-12} + 1.0025^{-36}} = \$13,939.00$$

4. *Fred takes out a 20 year mortgage for \$150,000 at $j_2 = 6\%$. This results in monthly payments of \$1,081.56, and a final payment of \$1,081.31. After 5 years, the mortgage is refinanced at $j_2 = 5\%$ for the remaining 15 years. The new monthly payments are \$995.30, with a final payment of \$994.83.*

Calculate the outstanding balance 3 years later, i.e. 8 years from the start of the mortgage.

We use the prospective method: The discounted value of the remaining 12 years of payments is given as

$$995.30\frac{1-1.025^{-24}}{1.025^{\frac{1}{6}}-1}-0.47\times 1.025^{-24}=\$107,912.53$$

Using the retrospective method: The value of the loan after 5 years is $150000\times 1.03^{10}=201,587.45....$ After 8 years, it is \$233,779.65. Meanwhile the accumulated value of the first 5 years of payments after 5 years, is $1081.56\frac{1.03^{10}-1}{1.03^{\frac{1}{6}}-1}=75,317.81...$, and after 8 years is $75,317.81...\times 1.025^6=87,345.57$. The accumulated value of the payments from the 5th to the 8th year is $995.30\frac{1.025^6-1}{1.025^{\frac{1}{6}}-1}=38541.74$, so the outstanding balance is $233779.65-87,345.57-38541.74=\$107,892.34$.

[The reason for the difference is a mistake in the payments that I calculated when setting the question. The correct payments after 5 years should have been \$995.17, with a final payment of \$994.07. If these correct payments were used, the two ways of calculating the answer would have given the same answer.]

5. *Mrs. Williams invests \$200 every month into an investment fund. At the end of 8 years, the value of the fund is \$24,945.69. What is the annual effective rate of return?*

   We try values for the monthly return i:

$i=0.005$	\$24,565.71
$i=0.006$	\$25,861.65

   We can interpolate between these to get $i=0.005+0.001\frac{24945.69-24565.71}{25861.65-24565.71}=0.005293...$ This is a monthly rate. The annual effective rate is then given by $1.005293...^{12}-1=6.54\%$.

6. *Mr. Smith donates \$50,000 to his old university. He intends for the donation to be used to set up an annual prize to be awarded at the end of the year. He wishes the first prize to be \$3,000 and would like the prizes to increase by 3% each year. If his money can be invested at $j_1=7\%$, how long can the prizes be awarded from this fund?*

   We have

$$\begin{aligned}50000&=3000\frac{1-\left(\frac{1.03}{1.07}\right)^n}{0.04}\\ \frac{2}{3}&=1-\left(\frac{1.03}{1.07}\right)^n\\ n&=\frac{\log\left(\frac{1}{3}\right)}{\log\left(\frac{1.03}{1.07}\right)}=28.835...\end{aligned}$$

So it could only last 28 years.

7. *Mrs. McNeil wants to borrow \$400,000 to set up her business. She has 2 options for repaying it over 5 years:*

   (i) borrow at $j_{12} = 10\%$ and amortise the debt with monthly payments. (ii) borrow at $j_{12} = 9\%$, repay the interest every month and set up a sinking fund which earns $j_{12} = 6\%$ to repay the principal.

   Which option is better (leads to lower monthly repayments)?

   If she borrows at 10%, the monthly payments are given by $400000 = R\frac{1-(1+\frac{0.1}{12})^{-60}}{\frac{0.1}{12}}$, which gives R=\$8,498.82 (rounded up).

   For the sinking fund method, the monthly interest is $400000 \times 0.0075 = \$3,000$, and the sinking fund deposits are obtained by solving $400000 = R\frac{1.005^{60}-1}{0.005}$, which gives $R = 5,733.12$ (rounded up), so the total monthly expense is \$8,733.12, so the amortisation method is better.