

MATH 2600/STAT 2600, Theory of Interest  
 FALL 2013  
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 Homework Sheet 5  
 Model Solutions

1. A company are considering a project. The project has the following expected cash flows (all amounts are calculated at the beginning of the year):

Year	0	1	2	3	4	5	6	7	8	9
<i>Net Cash Flow (000)</i>	-100	10	20	20	-10	20	20	20	30	30

- (a) What is the Net Present value of the project at  $j_1 = 3\%$ ?

At  $j_1 = 3\%$ , the net present value is  $-100 + 10(1.03)^{-1} + 20(1.03)^{-2} + 20(1.03)^{-3} - 10(1.03)^{-4} + 20(1.03)^{-5} + 20(1.03)^{-6} + 20(1.03)^{-7} + 30(1.03)^{-8} + 30(1.03)^{-9} = 34.92$ .

- (b) What is the Net Present value of the project at  $j_1 = 11\%$ ?

At  $j_1 = 11\%$ , the net present value is  $-100 + 10(1.11)^{-1} + 20(1.11)^{-2} + 20(1.11)^{-3} - 10(1.11)^{-4} + 20(1.11)^{-5} + 20(1.11)^{-6} + 20(1.11)^{-7} + 30(1.11)^{-8} + 30(1.11)^{-9} = -9.78$ .

- (c) What is the internal rate of return?

We compute the net present value for a few interest rates:

$j_1$	Net Present Value
11.000%	-9.78
3.000%	34.92
10.000%	-5.49
8.500%	1.52
9.000%	-0.90
8.800%	0.06
8.850%	-0.18
8.815%	-0.01
8.805%	0.04

So the internal rate of return is 8.81%.

2. A company are considering two projects. The projects have the following expected cash flows (all amounts are calculated at the beginning of the year):

Year	0	1	2	3	4	5	6	7	8
<i>Project 1 Net Cash Flow (000)</i>	-150	10	10	20	30	40	40	40	20
<i>Project 2 Net Cash Flow (000)</i>	-70	30	10	10	20	10	20	10	-30

(a) Which project should they prefer if the cost of capital is  $j_1 = 2\%$ ?

At  $j_1 = 2\%$ , the first project has net present value  $-150 + 10(1.02)^{-1} + 10(1.02)^{-2} + 20(1.02)^{-3} + 30(1.02)^{-4} + 40(1.02)^{-5} + 40(1.02)^{-6} + 40(1.02)^{-7} + 20(1.02)^{-8} = 39.62$ , while the second has net present value  $-70 + 30(1.02)^{-1} + 10(1.02)^{-2} + 10(1.02)^{-3} + 20(1.02)^{-4} + 10(1.02)^{-5} + 20(1.02)^{-6} + 10(1.02)^{-7} - 30(1.02)^{-8} = 6.84$ , so they should prefer the first project.

(b) Which project should they prefer if the cost of capital is  $j_1 = 12\%$ ?

If the cost of capital is  $j_1 = 12\%$ , the net present value of the first project is  $-150 + 10(1.12)^{-1} + 10(1.12)^{-2} + 20(1.12)^{-3} + 30(1.12)^{-4} + 40(1.12)^{-5} + 40(1.12)^{-6} + 40(1.12)^{-7} + 20(1.12)^{-8} = -30.66$ , while net present value of the second is  $-70 + 30(1.12)^{-1} + 10(1.12)^{-2} + 10(1.12)^{-3} + 20(1.12)^{-4} + 10(1.12)^{-5} + 20(1.12)^{-6} + 10(1.12)^{-7} - 30(1.12)^{-8} = -7.20$ , so they should prefer the second project (or neither project).

3. Mrs. Butler has \$200,000 in her investment fund at the start of the year. One month later, she withdraws \$130,000. Another 3 months later (4 months from the start of the year), she withdraws a further \$80,000 (there is enough money in the fund to cover this withdrawal). After another 5 months, (9 months from the start of the year), she wins the lottery and invests \$3,000,000 in the fund. At the end of the year, her investment advisor proudly tells her that her dollar-weighted return for the year is 11%. How much does she have in the fund at the end of the year?

At 11% simple interest, the values at the end of the year are  $200000 \times 1.11 - 130000 \times 1.10080 - 80000 \times 1.07333333 + 3000000 \times 1.0275 = 3075525.00$ .

4. Dr. Campbell is managing an investment fund. At the start of the year, there is \$7,000,000 in the fund. The fund activity is summarised in the following table:

Months from start of year	Fund value before transaction	Net Deposit	Fund value after transaction
0			7,000,000
1	7,020,000	-100,000	6,920,000
3	7,250,000	500,000	7,750,000
4	7,720,000	-100,000	7,620,000
5	7,830,000	600,000	8,430,000
6	7,660,000	-200,000	7,460,000
8	7,540,000	500,000	8,040,000
10	7,220,000	-100,000	7,120,000
12	7,420,000		7,420,000

Calculate her time-weighted rate of return for the year.

Her time-weighted rate of return is given by

$$\left(\frac{7020000}{7000000}\right) \left(\frac{7250000}{6920000}\right) \left(\frac{7720000}{7750000}\right) \left(\frac{7830000}{7620000}\right) \left(\frac{7660000}{8430000}\right) \left(\frac{7540000}{7460000}\right) \left(\frac{7220000}{8040000}\right) \left(\frac{7420000}{7120000}\right)$$

$$= 0.9243$$

So her time-weighted rate of return is  $-7.57\%$ .

5. A company has an account which pays interest at  $j_1 = 2.5\%$  on credit balances, and charges interest at  $j_1 = 6\%$  on debit balances. The company is considering using this account to fund possible projects with the following expected cashflows: (All amounts are calculated at the beginning of the year.)

Year	0	1	2	3	4	5	6	7
Project 1 Net Cash Flow (000)	-200	10	20	30	40	50	50	50
Project 2 Net Cash Flow (000)	-100	30	10	10	30	50	20	10

How much money does the company have in its account at the end of 7 years if it invests in:

- (a) The first project?

If it invests in the first project, it has the following balances at the end of each year:

Years Passed	Balance
0	-200
1	-202
2	-194.12
3	-175.77
4	-146.31
5	-105.09
6	-61.40
7	-15.08

- (b) The second project?

If it invests in the second project, it has the following balances at the end of each year:

Years Passed	Balance
0	-100
1	-76
2	-70.56
3	-64.79
4	-38.68
5	9.00
6	29.22
7	39.95

- (c) Both projects?

If it invests in both projects, it has the following balances at the end of each year:

Years Passed	Balance
0	-300
1	-278
2	-264.68
3	-240.56
4	-184.99
5	-96.09
6	-31.86
7	26.23