MATH 2600/STAT 2600, Theory of Interest
FALL 2013
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Homework Sheet 6
Model Solutions

1. *The current term structure has the following yields on zero-coupon bonds:*

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
rate	*3%*	*3%*	*3.5%*	*4.5%*	*5%*	*5.6%*	*5.8%*	*6%*

How much should be paid for a \$100 face-value bond with semi-annual coupons, maturing at par in 4 years time, with:

(a) 2% annual coupon rate?

The bond has semiannual coupons for \$1. The present value is given by $1.015^{-1} + 1.015^{-2} + 1.0175^{-3} + 1.0225^{-4} + 1.025^{-5} + 1.028^{-6} + 1.029^{-7} + 101 \times 1.03^{-8} = 86.10$

(b) 7% annual coupon rate?

The bond has semiannual coupons for \$3.5. The present value is given by $3.5 \times 1.015^{-1} + 3.5 \times 1.015^{-2} + 3.5 \times 1.0175^{-3} + 3.5 \times 1.0225^{-4} + 3.5 \times 1.025^{-5} + 3.5 \times 1.028^{-6} + 3.5 \times 1.029^{-7} + 103.5 \times 1.03^{-8} = 104.00$

(c) 12% annual coupon rate?

The bond has semiannual coupons for \$6. The present value is given by $6 \times 1.015^{-1} + 6 \times 1.015^{-2} + 6 \times 1.0175^{-3} + 6 \times 1.0225^{-4} + 6 \times 1.025^{-5} + 6 \times 1.028^{-6} + 6 \times 1.029^{-7} + 106 \times 1.03^{-8} = 121.90.$

2. What are the yields to maturity for the bonds in Q.1 (a) and (b)?

In (a), we try a few values for interest rate to find:

j_2	Price
5.000%	89.24
5.500%	87.58
5.800%	86.60
5.900%	86.28
5.950%	86.12
5.965%	86.07
5.955%	86.11

So the yield to maturity is 5.96%.

Similarly for (b):

j_2	Price
5.500%	105.32
5.700%	104.59
5.900%	103.87
5.865%	103.995
5.855%	104.03

So the yield to maturity is 5.86%.

3. *For the term structure in Q. 1, which of the one-year periods whose forward rates can be determined from the spot rates given has the largest implied forward rate?*

   The implied forward rates are:

Years to start of period	1-year interest rate
0	$1.015^2 - 1 = 3.02\%$
0.5	$\frac{1.0175^3}{1.015} - 1 = 3.79\%$
1	$\frac{1.0225^4}{1.015^2} - 1 = 6.06\%$
1.5	$\frac{1.025^5}{1.0175^3} - 1 = 7.40\%$
2	$\frac{1.028^6}{1.0225^4} - 1 = 7.970\%$
2.5	$\frac{1.029^7}{1.025^5} - 1 = 7.966\%$
3	$\frac{1.03^8}{1.028^6} - 1 = 7.33\%$

   So the period starting in two years time gives the largest forward rate.

4. *The spot-rates for 1, 2 and 3 year strip bonds are 5%, 5.3% and 5.6% annually. You have the opportunity to borrow or lend money at these rates, and you also have the opportunity to arrange to borrow money in two year's time for one year, at an annual rate of 6.4%, or to lend money in two years time for one year at an annual rate of 6.2%. Can you construct an arbitrage possibility?*

   The implied forward rate for a one-year loan starting in two years' time is $\frac{1.056^3}{1.053^2} - 1 = 6.203\%$. Since you do not have an opportuinity to lend at a higher rate than this, or to borrow at a lower rate, it is not possible to set up an arbitrage possibility.

5. *Mr and Mrs. Dunn are borrowing $150,000 at a variable rate of prime+2.3%. They are making interest-only payments annually. They make a forward rate agreement with the bank, so that the interest rate for the third year (starting two years from now) will be 7%. In two years time, the prime rate is 2%. How much money do they need to pay the bank*

   (a) If the payment is due at the begining of the year?

   At a rate of prime+2.3%, their interest payment is 4.3%, or $6,450 at the end of the year. The forward rate agreement makes their payment equal to 7%, or $10,500 at the end of the year. They would therefore need to pay the bank \$4,050 at the end of the year. The present value of this amount is $4050(1.043)^{-1} = 3883.03$.

(b) If the payment is due at the end of the year?

At a rate of prime+2.3%, their interest payment is 4.3%, or \$6,450 at the end of the year. The forward rate agreement makes their payment equal to 7%, or \$10,500 at the end of the year. They therefore need to pay the bank the difference, which is \$4,050.

6. *Mr. Evans can borrow at 4% on the fixed-rate market, or at prime+1.5% on the variable rate market. Mrs. French can borrow at 6% on the fixed-rate market, or at prime+2.8% on the variable rate market. Mr. Garner offers to lend Mr. Evans \$400,000 at prime+1.2% and to lend Mrs. French \$400,000 at 5.7%. How much spread income does Mr. Garner make on this transaction?*

   Mr. Evans can borrow \$400,000 on the fixed rate market at 4%, and Mrs. French can borrow \$400,000 on the variable rate market at prime+2.8%, so between them, they can borrow \$800,000 with interest payments of \$400,000 times (prime+6.8%). Mr. Garner is offering to lend them a combined total of \$800,000, with interest payments of \$400,000 times (prime+6.9%), so the difference is $\$400,000 \times 0.1\% = \$400$ per year spread income.

7. *The current term structure has the following yields on zero-coupon bonds:*

Term(years)	1	2	3	*4*	5	*6*
rate	*4.5%*	*5.1%*	*5.3%*	*5.5%*	*5.6%*	*5.6%*

   Mrs. Harper has a floating rate loan of \$800,000, with annual interest-only payments. She wishes to exchange this for a fixed rate over the next 6 years (i.e. she wants to pay the same interest rate over the next 6 years). What should this rate be?

   The implied forward rates, and interest payments are given in the following table:

Years to start of period	1-year interest rate	Interest payment	Present value of interest payment
0	4.5%	\$36,000	$36000(1.045)^{-1} = 34449.76$
1	$\frac{1.051^2}{1.045} - 1 = 5.70\%$	45627.56	41306.82
2	$\frac{1.053^3}{1.051^2} - 1 = 5.70\%$	45609.14	39063.11
3	$\frac{1.055^4}{1.053^3} - 1 = 6.10\%$	48818.26	39406.91
4	$\frac{1.056^5}{1.055^4} - 1 = 6.00\%$	48007.59	36558.66
5	$\frac{1.056^6}{1.056^5} - 1 = 5.6\%$	44800.00	32306.84

   The total present value of these payments is therefore \$223,092.11. She therefore needs to replace these with equal payments of X so that the present value of these payments is also \$223,092.11. The present value of the payments of X is $X(1.045^{-1}+1.051^{-2}+1.053^{-3}+1.055^{-4}+1.056^{-5}+$

1.056^{-6}), so we get $X = \frac{223092.11}{1.045^{-1}+1.051^{-2}+1.053^{-3}+1.055^{-4}+1.056^{-5}+1.056^{-6}} = 44541.92$.

8. *For the spot rates from Q. 7, what is the at-par yield of a 6-year bond with annual coupons?*

   At a coupon rate r, a bond with face value 100 has value $100r(1.045)^{-1} + 100r(1.051)^{-2}+100r(1.053)^{-3}+100r(1.055)^{-4}+100r(1.056)^{-5}+100r(1.056)^{-6}+ 100(1.056)^{-6}$. We need to find the r such that this is 100. This is given by

$$r = \frac{100 - 100(1.056)^{-6}}{100(1.045^{-1} + 1.051^{-2} + 1.053^{-3} + 1.055^{-4} + 1.056^{-5} + 1.056^{-6})} = 5.57\%$$