MATH 2600/STAT 2600, Theory of Interest
FALL 2014
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Homework Sheet 4
Model Solutions

1. *Calculate the price that should be paid for each of the following bonds to obtain the desired yield:*

   (a) Face value \$150,000, maturing at par in 8 years, coupon rate $j_2 = 3\%$, desired yield $j_2 = 4\%$.

   Using Makeham's formula, $K = 150000(1.02)^{-16} = \$109,266.87$, so $P = 109266.87 + (150000 - 109266.87)\frac{3}{4} = \$139,816.72$.

   (b) Face value \$110,000, maturing at par in 8 years, coupon rate $j_2 = 6\%$, desired yield $j_2 = 4\%$.

   Using Makeham's formula, $K = 110000(1.02)^{-16} = \$80,129.04$, so $P = 80129.04 + (110000 - 80129.04)\frac{6}{4} = \$124,935.48$.

2. *At what interest rate would the two bonds in Question 1 have the same present value?*

   (i) $j_2 = 16.10\%$

   (ii) $j_2 = 17.43\%$

   (iii) $j_2 = 19.82\%$

   (iv) $j_2 = 21.35\%$

   The differences between the payments are that the coupon payments are \$2,250 for the first bond and \$3,300 for the second bond, so the difference in coupon payments is \$1,050, while the difference in the final payment is \$40,000. The bonds have the same present value if the net value of these differences is zero. That is if $1050s_{\overline{16}|i} = 40000$, or $\frac{(1+i)^{16}-1}{i} = \frac{40000}{1050} = 38.095$.

   We try the values given:

j_2	i	$s_{\overline{16}\|i}$
16.10%	8.05%	30.452
17.43%	8.715%	32.214
19.82%	9.91%	35.673
21.35%	10.675%	38.095

   So the two bonds have the same present value at $j_2 = 21.35\%$.

3. *(a) Write out a complete bond amortisation schedule for a bond with face value \$20,000 with coupon rate $j_2 = 3\%$, maturing at par in 4 years, sold to an investor who wishes to receive a yield of $j_2 = 7\%$*

We calculate the amount payed using Makeham's formula. $K = 20000(1.035)^{-8} = \$15,188.23$, so $P = 15188.23 + (20000 - 15188.23)\frac{3}{7} = \$17,250.42$.

Coupon Number	Book Value	Coupon	Interest	Principal Repaid
1	\$17,250.42	\$300.00	\$603.76	$\$ - 303.75$
2	\$17,554.17	\$300.00	\$614.40	$\$ - 314.39$
3	\$17,868.56	\$300.00	\$625.40	$\$ - 325.39$
4	\$18,193.95	\$300.00	\$636.79	$\$ - 336.78$
5	\$18,530.73	\$300.00	\$648.58	$\$ - 348.57$
6	\$18,879.30	\$300.00	\$660.78	$\$ - 360.77$
7	\$19,240.07	\$300.00	\$673.40	$\$ - 373.39$
8	\$19,613.46	\$20300.00	\$686.47	$\$19,613.46$

(b) Write out a complete bond amortisation schedule for a bond with face value \$25,000 with coupon rate $j_2 = 5.5\%$, maturing at par in 4 years, sold to an investor who wishes to receive a yield of $j_2 = 3\%$

We calculate the amount payed using Makeham's formula. $K = 25000(1.015)^{-8} = \$22,192.78$, so $P = 22192.78 + (25000 - 22192.78)\frac{5.5}{3} = \$27,339.35$.

Coupon Number	Book Value	Coupon	Interest	Principal Repaid
1	\$27,339.35	\$687.50	\$410.09	\$277.41
2	\$27,061.94	\$687.50	\$405.93	\$281.57
3	\$26,780.37	\$687.50	\$401.71	\$285.79
4	\$26,494.58	\$687.50	\$397.42	\$290.08
5	\$26,204.50	\$687.50	\$393.07	\$294.43
6	\$25,910.07	\$687.50	\$388.65	\$298.85
7	\$25,611.22	\$687.50	\$384.17	\$303.33
8	\$25,307.89	\$25,687.50	\$379.62	\$25,307.88

4. *A bond has face value \$24,000, maturity in 12 years, coupon rate $j_2 = 5\%$. After 2 years and 8 months, it is sold to a bank, who wish to receive a yield of 5.8%. Calculate*

   (a) The flat price.

   Using Makeham's formula to get the price after 2 years and 6 months, we see that $K = 24000(1.029)^{-19} = 13941.81$, so $P = 13941.81 + (24000 - 13941.81)\frac{5}{5.8} = 22612.66$. The flat price is this price two months later — that is $P = 22612.66(1.029)^{\frac{1}{3}} = \$22,829.17$.

   (b) The quoted price.

   The accrued interest is one third of the coupon payment, or $\frac{1}{3} \times 24000 \times 0.025 = \$200$, so the quoted price is $22829.17 - 200 = \$22,629.17$.

5. *Mrs. Wood buys a bond with face value \$65,000, maturing at par in 13 years, with coupon rate 2%, for a price to yield $j_2 = 4.1\%$. After 4 years, interest rates decrease, and she sells the bond to Mr. Young, who wishes to receive a yield of $j_2 = 3.7\%$.*

   (a) What is Mrs. Wood's rate of return?

(i) $j_2 = 4.22\%$

(ii) $j_2 = 4.88\%$

(iii) $j_2 = 5.06\%$

(iv) $j_2 = 5.25\%$

We use Makeham's formula: For the price Mrs. Wood pays, $K = 65000(1.0205)^{-26} = 38350.86$, so $P = 38350.86 + (65000 - 38350.86)\frac{2}{4.1} = \$51,350.44$.

For the price Mr. Young pays, $K = 65000(1.0185)^{-18} = \$46,732.04$, so $P = 46732.04 + (65000 - 46732.04)\frac{2}{3.7} = \$56,606.61$.

Mrs. Wood invests \$51,350.44, receives 8 coupon payments of \$650, and a final payment of \$56,606.61 4 years later. For each of the given rates of return, we calculate the present value of the payments she receives — that is $56606.61(1+i)^{-8} + 650a_{\overline{8}|i}$:

j_2	i	Present value of payments received.
4.22%	2.11%	\$52,637.46
4.88%	2.44%	\$51,350.36
5.06%	2.53%	\$51,005.67
5.25%	2.625%	\$50,644.74

(b) How much would interest rates need to have decreased for Mrs. Wood to achieve a $j_2 = 6.2\%$ *rate of return?*

(i) $j_2 = 2.89\%$

(ii) $j_2 = 3.03\%$

(iii) $j_2 = 3.24\%$

(iv) $j_2 = 3.47\%$

To achieve a $j_2 = 6.2\%$ rate of return, she would have to sell the bond for $51350.44(1.031)^8 - 650s_{\overline{8}|0.031} = \$59,755.59$. If she sells it for this price, we need to calculate the buyer's rate of return. We calculate the price the buyer would pay at the given interest rates:

j_2	i	Price of bond.
2.89%	1.445%	\$60,444.28
3.03%	1.515%	\$59,760.66
3.24%	1.62%	\$58,751.39
3.47%	1.735%	\$57,667.83

So interest rates need to have dropped to $j_2 = 3.03\%$ for her to make this rate of return.