MATH 2600/STAT 2600, Theory of Interest
FALL 2014
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Homework Sheet 6
Model Solutions

1. *The current term structure has the following semi-annual yields on zero-coupon bonds:*

Term(years)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
rate	*1%*	*1.4%*	*1.5%*	*2.5%*	*2.7%*	*2.8%*	*2.8%*	*2.9%*

How much should be paid for a \$100 face-value bond with semi-annual coupons, maturing at par in 4 years time, with:

(a) 2% annual coupon rate?

With 2% annual coupon rate, a \$100 face-value bond has 8 coupons of \$1, and a final payment of \$100, for a present value of $1(1.005)^{-1} + 1(1.007)^{-2} + 1(1.0075)^{-3} + 1(1.0125)^{-4} + 1(1.0135)^{-5} + 1(1.014)^{-6} + 1(1.014)^{-7} + 1(1.0145)^{-8} + 100(1.0145)^{-8} = 7.56 + 89.12 = \$96.69$.

(b) 6% annual coupon rate?

With 6% annual coupon rate, a \$100 face-value bond has 8 coupons of \$3, and a final payment of \$100, for a present value of $3(1.005)^{-1} + 3(1.007)^{-2} + 3(1.0075)^{-3} + 3(1.0125)^{-4} + 3(1.0135)^{-5} + 3(1.014)^{-6} + 3(1.014)^{-7} + 3(1.0145)^{-8} + 100(1.0145)^{-8} = 3 \times 7.56 + 89.12 = \$111.81$.

(c) 11% annual coupon rate?

With 11% annual coupon rate, a \$100 face-value bond has 8 coupons of \$5.5, and a final payment of \$100, for a present value of $5.5(1.005)^{-1} + 5.5(1.007)^{-2} + 5.5(1.0075)^{-3} + 5.5(1.0125)^{-4} + 5.5(1.0135)^{-5} + 5.5(1.014)^{-6} + 5.5(1.014)^{-7} + 5.5(1.0145)^{-8} + 100(1.0145)^{-8} = 5.5 \times 7.56 + 89.12 = \$130.72$.

2. *What are the yields to maturity for the bonds in Q.1 (a) and (b)?*

(i) $j_2 = 2.74\%$

(ii) $j_2 = 2.80\%$

(iii) $j_2 = 2.85\%$

(iv) $j_2 = 2.88\%$

For Q. 1(a), the price for a yield of $j_2 = 2i$ is given by $K = 100(1+i)^{-8}$, then $P = K + (100 - K)\frac{1}{i}$. For Q. 1(b), the value of K is the same, and the price is given by $P = K + (100 - K)\frac{3}{i}$. We calculate the prices for the yields given:

j_2	i	Price for (a)	Price for (b)
2.74%	1.37%	\$97.21	\$112.27
2.80%	1.40%	\$96.99	\$112.03
2.85%	1.425%	\$96.81	\$111.83
2.88%	1.44%	\$96.70	\$111.71

So the yields are $j_2 = 2.88\%$ and $j_2 = 2.85\%$ respectively.

3. *For the term structure in Q. 1, which of the one-year periods whose forward rates can be determined from the spot rates given has the largest implied forward rate?*

   Starting now, the one-year implied forward rate is $(1.0075)^2 - 1 = 1.51\%$.

   Starting in $\frac{1}{2}$ year, the implied forward rate is $\frac{(1.0125)^3}{1.005} - 1 = 3.28\%$.

   Starting in 1 year, the implied forward rate is $\frac{(1.0135)^4}{1.007^2} - 1 = 4.05\%$.

   Starting in $1\frac{1}{2}$ years, the implied forward rate is $\frac{(1.014)^5}{1.0075^3} - 1 = 4.82\%$.

   Starting in 2 years, the implied forward rate is $\frac{(1.014)^6}{1.0125^4} - 1 = 3.43\%$.

   Starting in $2\frac{1}{2}$ years, the implied forward rate is $\frac{(1.0145)^7}{1.0135^5} - 1 = 3.43\%$.

   So the period starting in $1\frac{1}{2}$ years has the highest implied forward rate.

4. *The spot-rates for 1, 2 and 3 year strip bonds are 3%, 3.5% and 3.6% annually. You have the opportunity to borrow or lend money at these rates, and you also have the opportunity to arrange to borrow money in two year's time for one year, at an annual rate of 3.6%, or to lend money in two years time for one year at an annual rate of 3.4%. Can you construct an arbitrage possibility?*

   The implied forward rate for a 1-year loan starting in two years time is $\frac{1.036^3}{1.035^2} - 1 = 3.80\%$. Since you can borrow money for less than this, you can arrange an arbitrage as follows:

   - Sell a 2-year strip-bond with face value \$1,000,000 for $1000000(1.035)^{-2} = \$933,510.70$.
   - Buy a 3-year strip bond for this money, with face value \$1,038,002.90.
   - Arrange to borrow \$1,000,000 in two years' time for one year at 3.6%. Use this to pay off the 2-year strip bond.
   - After 3 years, you will owe \$1,036,000 for the forward loan, and you will receive \$1,038,002.90 from the strip bond. This leaves you an arbitrage profit of \$2,002.90.

5. *Mrs. Bale is borrowing \$350,000 at a variable rate of prime+1.3%. She is making interest-only payments annually. She makes a forward rate agreement with the bank, so that the interest rate for the third year (starting two years from now) will be 5%. In two years time, the prime rate is 2%. How much money does she need to pay the bank*

(a) If the payment is due at the end of the year?

Mrs. Bale has agreed that her net payment at the end of the third year will be $350000(0.05) = \$17,500$. The rate on her variable rate loan is 3.3%, so she owes $350000(0.033) = \$11,550$ on this. She needs to pay the remainder to the bank. That is $17500 - 11550 = \$5,950$ at the end of the year.

(b) If the payment is due at the begining of the year?

Since the interest rate for the year is 3.3%, a payment of \$5,950 at the end of the year is equivalent to a payment of $5950(1.033)^{-1} = \$5,759.92$ at the beginning of the year.

6. *Mr. and Mrs. Chapman can borrow at 3% on the fixed-rate market, or at prime+1.8% on the variable rate market. Mr. Dodd can borrow at 5% on the fixed-rate market, or at prime+3.5% on the variable rate market. Mrs. Ellerman arranges swaps with both of them so that Mr. and Mrs. Chapman can borrow \$600,000 at prime+1.7% and Mr. Dodd can borrow \$600,000 at 4.9%. How much spread income does Mrs. Ellerman make on this transaction?*

   The total interest paid by the Chapmans and Mr. Dodd on this transaction is prime+6.6% on \$600,000, while if the Chapmans borrowed on the fixed rate market at 3%, and Mr. Dodd borrowed on the variable rate market at prime+3.5%, the total interest paid would be prime+6.5%. The spread income is the difference between these amounts. That is 0.1% of \$600,000 or \$600 per year.

7. *The current term structure has the following annual yields on zero-coupon bonds:*

Term(years)	1	2	3	*4*	5	*6*	*7*	*8*
rate	*4.8%*	*4.7%*	*5.2%*	*5.4%*	*5.6%*	*5.7%*	*5.6%*	*5.7%*

   Mrs. Foley has a floating rate loan of \$1,200,000, with annual interest-only payments. She wishes to exchange this for a fixed rate over the next 8 years (i.e. she wants to pay the same interest rate over the next 8 years). What should this rate be?

   After 8 years, the \$1,200,000 that she still owes has a present value of $1200000(1.057)^{-8} = \$770,160.69$, so the present value of all the interest payments that she makes is $1200000 - 770160.68 = \$429,839.31$. She wants to pay this by making payments of X every year. The value of X needs to satisfy $X(1.048)^{-1} + X(1.047)^{-2} + X(1.052)^{-3} + X(1.054)^{-4} + X(1.056)^{-5} + X(1.057)^{-6} + X(1.056)^{-7} + X(1.057)^{-8} = 429839.31$, so $X = \frac{429839.31}{(1.048)^{-1}+(1.047)^{-2}+(1.052)^{-3}+(1.054)^{-4}+(1.056)^{-5}+(1.057)^{-6}+(1.056)^{-7}+(1.057)^{-8}} = \$67,809.75$. As a percentage of the balance owed, we have $\frac{67809.75}{1200000} = 5.65\%$.

8. *For the spot rates from Q. 7, what is the at-par yield of a 7-year bond with annual coupons?*

   We are looking for the coupon rate r so that a 7-year bond with coupon rate r would be purchased at par. The price for a \$100 bond with coupon rate r is $100(1.056)^{-7}+r(1.056)^{-7}+r(1.057)^{-6}+r(1.056)^{-5}+r(1.054)^{-4}+r(1.052)^{-3}+r(1.047)^{-2}+r(1.048)^{-1}$. We want this to equal 100. That is, we want $r((1.056)^{-7}+(1.057)^{-6}+(1.056)^{-5}+(1.054)^{-4}+(1.052)^{-3}+(1.047)^{-2}+(1.048)^{-1}) = 100-100(1.056)^{-7}$, so $r = \frac{100-100(1.056)^{-7}}{(1.056)^{-7}+(1.057)^{-6}+(1.056)^{-5}+(1.054)^{-4}+(1.052)^{-3}+(1.047)^{-2}}$
   5.57%.