MATH 3030, Abstract Algebra
FALL 2012
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Homework Sheet 2
Due: Friday 5th October: 3:30 PM

Basic Questions

1. (a) Show that the collection of symmetries of a regular hexagon is a group of order 12.

   (b) Find all subgroups of this group.

2. How many elements are in the subgroup of $\mathbb{Z}_{45}$ generated by 12?

3. Which of the following are subgroups of the group of permutations of the 6-element set $\{1, 2, 3, 4, 5, 6\}$.

   (a) The collection of permutations that fix the subsets $\{1, 2, 3\}$ and $\{4, 5\}$.

   (b) The collection of permutations that send the subset $\{1, 2\}$ to the subset $\{4, 5\}$.

4. Which of the following are subgroups of the additive group of real numbers:

   (a) The collection of real numbers greater than or equal to 0.

   (b) The collection of numbers whose decimal expansion terminates after a finite number of decimal places. [Technically, these numbers have two decimal expansions and only one terminates after a finite number of places.]

   (c) The collection of numbers x such that x^2 is a rational number.

5. How many generators does the cyclic group of order 28 have?

6. Draw the Cayley graph of $\mathbb{Z}_{15}$ with generators 9 and 10.

Theoretical Questions

7. Show that if a subgroup of the real numbers contains an interval $[a, b]$, with $a < b$ then it must be the whole group of real numbers.

8. H and K are subgroups of G. The union $H \cup K$ is also a subgroup of G. Prove that $H \subseteq K$ or $K \subseteq H$.

9. Show that a group with only finitely many subgroups is finite. [Hint: consider the cyclic subgroups generated by each element.]

10. The centre Z of a group G is the set of all elements in G that commute with all elements in G. That is $Z = \{a \in G | (\forall x \in G)(ax = xa)\}$. Prove that Z is a subgroup of G.

11. (a) If G is a group, and every finitely generated subgroup of G is cyclic, show that G is abelian.

    (b) must G be cyclic?