

MATH/STAT 3360, Probability
FALL 2011
Toby Kenney
Sample Midterm Examination

This Sample Midterm has more questions than the actual midterm, in order to cover a wider range of questions.

1. How many ways are there to choose 5 students from a class of 25?
2. There are n people and m books in a library, where $n < m$. Each person selects one book to read. In how many different ways can this be achieved?
3. How many distinct ways can the letters of the word "PERMUTATION" be arranged?
4. What is the probability that the sum of 3 fair 6-sided dice is 6?
5. What is the probability that a five-card poker hand is a four of a kind (has four cards of one rank and one card of another)?
6. A fair coin is tossed 7 times.
 - (a) What is the probability that the sequence HHHT occurs somewhere in the 7 tosses?
 - (b) What is the probability that the sequence THTH occurs somewhere in the 7 tosses?
7. five coins are tossed: are the following events independent?
 - (i) There are the same number of heads among the first two tosses and the last two tosses.
 - (ii) The total number of heads is 3.
8. Suppose the number of cars that want to park in a particular street each day is a Poisson random variable with parameter 6. There are 4 parking spaces on the street.
 - (a) What is the probability that the number of cars parking on that street is exactly 3?
 - (b) What is the probability that all the parking spots are taken?
 - (c) What is the expected number of free parking spaces?
9. A patient is given a routine test for a rare disease. The disease affects 3 people in 1000. The test is 95% accurate, so there is a 5% chance of giving the wrong result. The test result is positive (i.e. indicates the patient has the disease). What is the probability that the patient actually has the disease?

10. A company is conducting a survey. They want to determine the proportion of people who would buy their new product. If the true proportion is 30%, how many people do they need to survey so that the probability that their estimate is within 2% of the true value (i.e. between 28% and 32%) is at least 95%?
11. A printer contains a pair of rollers, between which a sheet of paper passes. One roller has diameter 5cm and the other has diameter 7cm. Each roller has a defect that affects 1cm of the circumference. When the paper is between both defects, it rips. A sheet of paper is 28cm long. What is the probability that a random sheet of paper gets ripped? [Hint: the positions of the rollers repeat every 5 rotations of the larger roller.]
12. A company makes light bulbs. The company has two machines for making them. light bulbs made by one machine have lifetime (in years) exponentially distributed with parameter 2, and light bulbs made by the other machine have lifetime exponentially distributed with parameter 3. 30% of its products are made by the first machine.
- (a) What is the probability that a randomly chosen light bulb lasts for at least 1 year?
- (b) Given that it lasts for 1 year, what is the probability that it was produced by machine 1?
- (c) Given that a light bulb has lasted for 1 year, what is the probability that it lasts for a second year?
13. A company produces 50,000,000 scratchcards, which it will sell for \$1 each. The scratchcards offer the following cash prizes:

Prize	Number of cards with this prize
0	48,000,000
\$10	1,850,000
\$100	140,000
\$1,000	9,750
\$10,000	235
\$100,000	14
\$1,000,000	1

- (a) What is the expected value and variance for the prize of a scratch card?
- (b) If the company sells 100,000 scratch cards, what is the expected profit?