ACSC/STAT 3703, Actuarial Models I
Further Probability with Applications to Actuarial Science
Winter 2015
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In Class Examples

Functions Used to Describe a Distribution

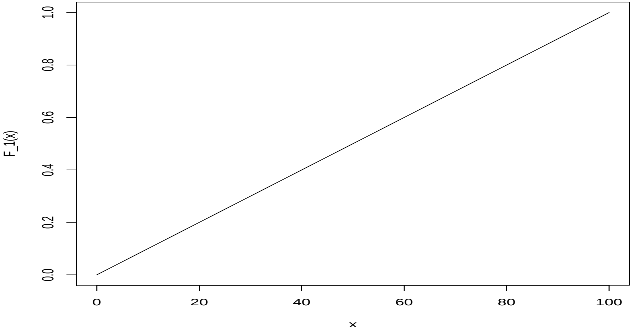
- Cumulative Distribution Function
- Survival Function

For continuous distributions:

- Probability Density Function
- Hazard Rate Functions (Force of Mortality)

Model I (Age at death)

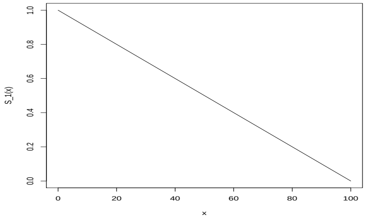
$$F_1(x) = \begin{cases} 0 & x < 0 \\ 0.01x & 0 \leqslant x < 100 \\ 1 & x \geqslant 100 \end{cases}$$



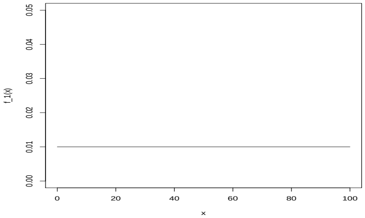
Question 1

Find the survival function, density function and hazard-rate function for Model I.

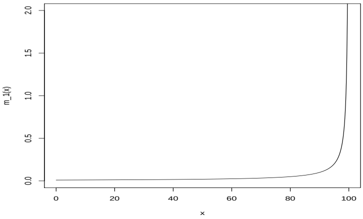
Model I, survival, pdf, hazard



Survival Function



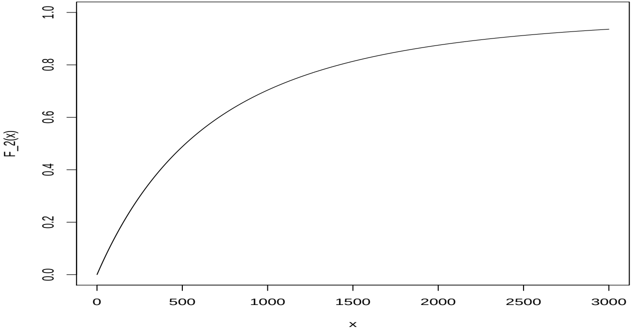
Density Function



Hazard Rate

Model II (Dollars paid on Insurance Claim)

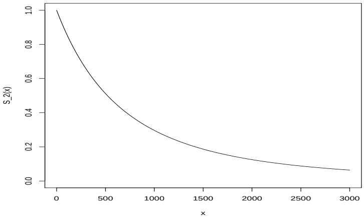
$$F_2(x) = \begin{cases} 0 & x < 0 \\ 1 - \left(\frac{2000}{x+2000}\right)^3 & x \geqslant 0 \end{cases}$$



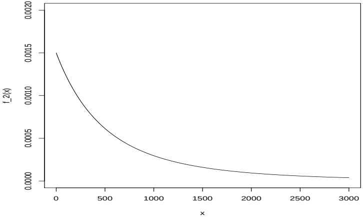
Question 2

Find the survival function, density function and hazard-rate function for Model II.

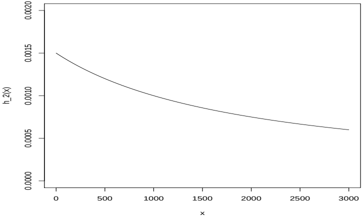
Model II, Survival, pdf, Hazard



Survival Function



Density Function



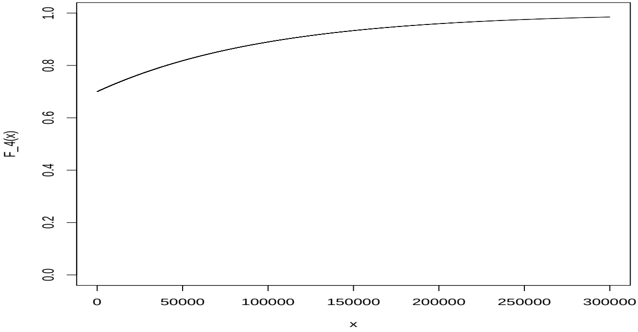
Hazard Rate

Model III (Number of Claims on a Policy in One Year)

$$F_3(x) = \begin{cases} 0 & x < 0 \\ 0.5 & 0 \leqslant x < 1 \\ 0.75 & 1 \leqslant x < 2 \\ 0.87 & 2 \leqslant x < 3 \\ 0.95 & 3 \leqslant x < 4 \\ 1 & x \geqslant 4 \end{cases}$$

Model IV (Amount Paid on an Insurance Policy in a Given Year)

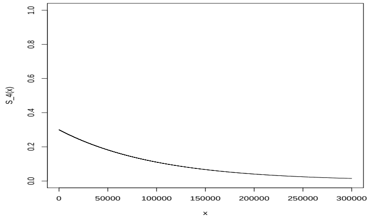
$$F_4(x) = \begin{cases} 0 & x < 0 \\ 1 - 0.3e^{-0.00001x} & x \geqslant 0 \end{cases}$$



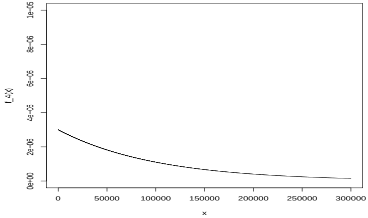
Question 3

Find the survival function, density function and hazard-rate function for Model IV.

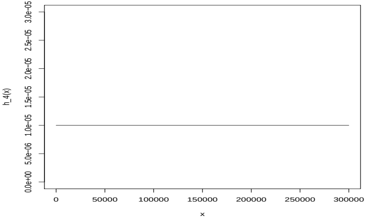
Model IV, Survival, pdf, Hazard



Survival Function



Density Function



Hazard Rate

Question 4

Find the coefficient of variation, skewness and kurtosis of models I–IV.

Question 5

A gamma distribution has probability density function given by

$$f(x) = \frac{\left(\frac{x}{\theta}\right)^{\alpha} e^{-\frac{x}{\theta}}}{x\Gamma(\alpha)}$$

where

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

Find the coefficient of variation and skewness of a gamma distribution.

Question 6

Calculate the mean excess loss function for model II.

Question 7

Find the median for a gamma distribution with parameters $\alpha = 2$ and $\theta = \frac{e^2}{6}$.

Question 8

Let X be a random variable, and let π_p denote the 100pth percentile of X. Calculate the 100pth percentiles of the excess loss random variable and limited loss random variable $X \wedge u$ in terms of π_p.

Question 9

A beta distribution has support $[0, 1]$ and probability density function

$$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx}$$

on its support. Suppose we want to construct a beta distribution such that $\alpha = 2$ and its 95th percentile is 0.8. What should β be?

3.3 Generating Functions and Sums of Random Variables

Question 10

An insurance company insures 16 companies. The claims from each company follow a gamma distribution with $\alpha = 1$ and $\theta = 250$. Calculate the probability that the total loss exceeds $6,000.

Question 11

A negative binomial random variable with parameters β and k has probability mass function given by

$$P(X = i) = \binom{r+i-1}{i}\beta^k(1+\beta)^{-(r+k)}$$

[r need not be an integer. The binomial coefficient is given by $\frac{r(r+1)\cdots(r+i-1)}{i!}$. If r is an integer. This is the distribution of the number of failures before r successes, if the probability of success is $p = \frac{1}{1+\beta}$.]

(a) Calculate the probability generating function for a negative binomial with parameters r and β.

(b) What is the distribution of the sum of negative binomial random variables with parameters r_1, β and r_2, β (same value of β)?

Question 12

A Pareto distribution with parameters α and θ has probability density function

$$f(x) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}}$$

Show that the kth moment exists only for $k < \alpha$.

Question 13

Let X have a Pareto distribution with parameters α and θ. Let Y have a Pareto distributions with parameters $\alpha' > \alpha$ and θ', chosen so that the distributions have the same mean. Show that for any $k > 1$ such that the kth moments of X and Y both exist, the kth moment of X is larger than the kth moment of Y.

Question 14

Let X have a normal distribution with mean μ and variance σ^2. Let Y have a gamma distribution with parameters α and θ. Calculate the limit $\lim_{x\to\infty} \frac{f_Y(x)}{f_X(x)}$. Which distribution has a heavier tail?

Question 15

Let X have a Pareto distribution with parameters α and θ, and let Y have a Pareto distribution with parameters α and θ'. Calculate the limit $\lim_{x \to \infty} \frac{f_Y(x)}{f_X(x)}$. Which distribution has a heavier tail?

Question 16

Calculate the limiting behaviour of the hazard rate functions for a gamma distribution and a Pareto distribution.

Question 17

Calculate the mean excess loss function for a Pareto distribution with parameters α and θ.

Question 18

Find the equilibrium distribution of a Pareto distribution with parameters α and θ.

Coherence Properties for Risk Measures

1. Subadditivity
2. Monotonicity
3. Positive homogeneity
4. Translation invariance

3.5 Measures of Risk

Coherence Properties for Risk Measures

1. Subadditivity
2. Monotonicity
3. Positive homogeneity
4. Translation invariance

Risk Measures

1. Standard deviation principle
2. VaR
3. TVaR

Question 19

Which of the coherence properties of risk measures does the standard deviation principle satisfy?

Question 20

The density function of an inverse Pareto distribution is

$$f(x) = \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}}$$

and the distribution function is

$$F(x) = \left(\frac{x}{x+\theta}\right)^{\tau}$$

Find the VaR and TVaR of this distribution at the level p.

Question 21

Show that TVaR is a coherent risk measure.

4.2 Simple vs. Complicated Models

Advantages of Simple Models

- Better estimation.
- Less variance in parameter estimates.

Advantages of Complicated Models

- More accurate description of true distribution.
- Less bias in parameter estimates.

Question 22

(a) Show that the following distributions are scale distributions:

- Exponential
- Gamma
- Normal
- Pareto

(b) Which of the above distributions have scale parameters?

Question 23

Claims follow a Pareto distribution with $\alpha = 2$. Claims experience uniform inflation of 6% per year. Let *r* be the ratio of claims exceeding *d* next year over the proportion of claims exceeding *d* this year. Calculate the limit of *r* as *d* increases to ∞.

Transformed Gamma Family

- Transformed Gamma (generalised gamma)

$$f_X(x) = \frac{\tau \left(\frac{x}{\theta}\right)^{\alpha\tau} e^{-\left(\frac{x}{\theta}\right)^{\tau}}}{x\Gamma(\alpha)}$$

- Gamma ($\tau = 1$)

$$f_X(x) = \frac{\left(\frac{x}{\theta}\right)^{\alpha} e^{-\left(\frac{x}{\theta}\right)}}{x\Gamma(\alpha)}$$

- Weibull ($\alpha = 1$)

$$f_X(x) = \frac{\tau \left(\frac{x}{\theta}\right)^{\tau} e^{-\left(\frac{x}{\theta}\right)^{\tau}}}{x}$$

- Exponential ($\tau = 1$, $\alpha = 1$)

$$f_X(x) = \frac{e^{-\frac{x}{\theta}}}{\theta}$$

Question 24

Let X be a mixture of 3 distributions:

- with probability p_1, it has mean μ_1 and variance ${\sigma_1}^2$.
- with probability p_2, it has mean μ_2 and variance ${\sigma_2}^2$.
- with probability p_3, it has mean μ_3 and variance ${\sigma_3}^2$.

What are the mean and variance of X?

Question 25

A car insurance company deals with three types of claim:

- 15% are for thefts. The claim amount in this case follows an inverse Pareto distribution with $\tau = 3$ and $\theta = 4000$.
- 75% are for collisions. The claim amount in this case follows a Gamma distributions with $\alpha = 50$ and $\theta = 100$.
- 10% are other claims. The claim amount in this case follows an exponential distribution with mean \$3,000.

What proportion of claims are over \$10,000?

Semiparametric and Nonparametric Distributions

Semiparametric Distributions

A variable-component mixture distribution is a mixture distribution where the number of components is not fixed in advance. Often all components of the mixture will have the same distribution (but different parameters).

Data Dependant Distributions

These have as many parameters as data points (or more). As the amount of available data increases, the model becomes more complicated to match.

- The empirical model
- Kernel smoothing models

Question 26

We obtain the following sample of claims (in thousands of dollars): 0.3, 1.2, 1.4, 1.9, 4.7. Using a kernel smoothing model with a Gaussian (normal) kernel with standard deviation 0.8, estimate the probability that the next claim received is more than 3.

Methods to Create New Distributions

This Chapter

- Multiplication by a constant
- Raising to a power
- Exponentiation
- Mixing
- Frailty models
- Splicing

More Detail Later

- Truncation
- Censorship
- Modification at Zero

Question 27

Let X have a beta distribution with parameters α and β. Find the probability density function of $5X$.

Question 28

Find the probability density function of the inverse of a Gamma distribution with parameters α and θ.

Question 29

If X follows a normal distribution with mean 0 and variance σ^2, what is the distribution of X^2?

Question 30

A log-normal distribution is the distribution of e^X, where X has a normal distribution with parameters μ and σ^2. Calculate the probability density function of a log-normal distribution.

Question 31

Let X follow a Pareto distribution with parameters α and θ. Let $Y = \log\left(1 + \frac{X}{\theta}\right)$. What is the distribution of Y?

Question 32

Suppose that the amount paid on a claim for an insurance policy on a car with value x follows an inverse gamma distribution with $\theta = 1000 + 0.1x$ and $\alpha = 5$. Suppose that the value of a randomly selected car follows an exponential distribution with mean $\$7,000$. What is the distribution of the amount paid on a random claim?

Question 33

Given a value of $\Theta = \theta$, the variable X follows an exponential distribution with hazard rate θ. Θ is uniformly distributed on $(1, 11)$. Calculate $S_X(0.5)$.

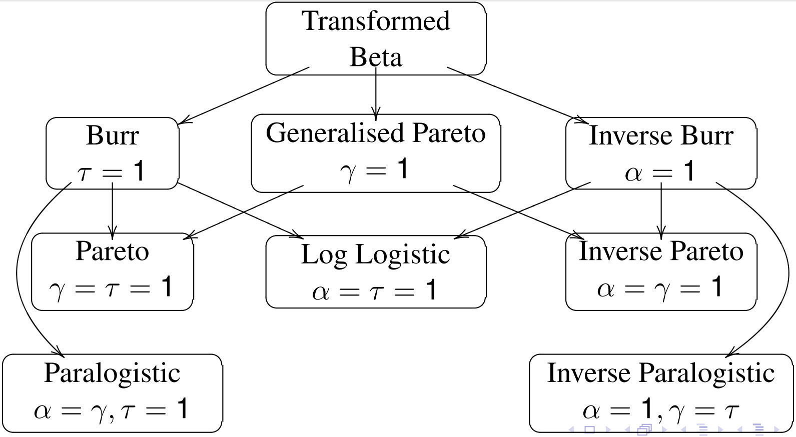
Question 34

The age at death of a random person is modelled as having a hazard rate given by $0.0001(20 - x)$ for people under the age of 10, then a hazard rate given by $10^{-5}x^2$ for people over the age of 10. Calculate the probability under this model that a randomly chosen person lives to age 75.

The Transformed Beta Family

Transformed Beta Distribution

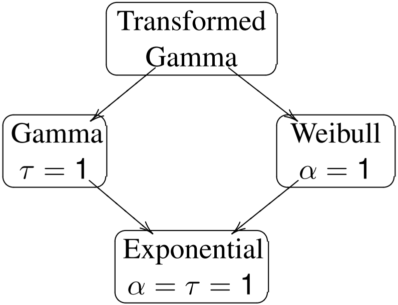
$$f_X(x) = \left(\frac{\Gamma(\alpha+\tau)}{\Gamma(\alpha)\Gamma(\tau)}\right) \frac{\gamma\left(\frac{x}{\theta}\right)^{\gamma\tau}}{x\left(1+\left(\frac{x}{\theta}\right)^{\gamma}\right)^{\alpha+\tau}}$$



The Transformed Gamma and Inverse Transformed Gamma Families

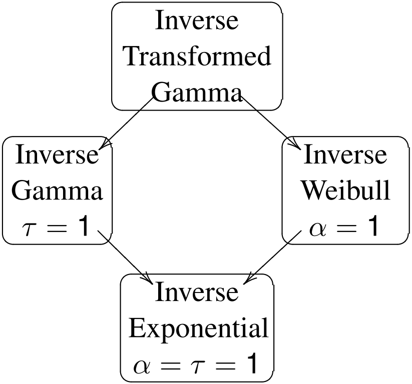
Transformed Gamma

$$f_X(x) = \frac{\tau\left(\frac{x}{\theta}\right)^{\alpha\tau} e^{-\left(\frac{x}{\theta}\right)^{\tau}}}{x\Gamma(\alpha)}$$



Inverse Transformed Gamma

$$f_X(x) = \frac{\tau\left(\frac{\theta}{x}\right)^{\alpha\tau} e^{-\left(\frac{\theta}{x}\right)^{\tau}}}{x\Gamma(\alpha)}$$



Question 35

Let X have a transformed beta distribution with parameters α, γ, θ and τ. Show that $\frac{1}{X}$ also follows a transformed beta distribution, and calculate its parameters.

Question 36

Show that the inverse transformed gamma distribution is the limit of a transformed beta distribution as $\tau \to \infty$, $\theta \to 0$ and $\theta\tau^{\frac{1}{\gamma}} \to \xi$.

Question 37

Show that the limiting distibution of the transformed gamma distribution as $\alpha \to \infty$, $\tau \to 0$ and $\theta \to 0$, with $\frac{\sqrt{\theta\tau}}{\tau} \to \sigma$ and $\frac{\theta^\tau \alpha - 1}{\tau} \to \mu$ is a lognormal distribution.

Question 38

Show that a distribution from the linear exponential family with pdf

$$f_X(x) = \frac{p(x)e^{r(\theta)x}}{q(\theta)}$$

has mean $\mu(\theta) = \frac{q'(\theta)}{r'(\theta)q(\theta)}$ and variance $\frac{\mu'(\theta)}{r'(\theta)}$.

Question 39

Find the mean and variance of a gamma distribution.

Question 40

(a) Calculate the probability generating function of a Poisson random variable.
(b) Show that a sum of independent Poisson random variables is a Poisson random variable.

Question 41

Let the number of losses experienced for a particular insurance product follow a Poisson distribution with parameter λ. Let each loss lead to a claim with probability p. Show that the number of claims follows a Poisson distribution with parameter λp, and that the number of losses which do not lead to claims is independent of the number of claims.

Question 42

An insurance company takes out 10 different reinsurance policies. Each one has probability 0.2 of leading to a claim. What is the probability that the company does not need to make any claims to the reinsurers?

6.3 Negative Binomial

Negative Binomial Distribution

$$P(X = k) = \binom{k + r - 1}{k} \left(\frac{\beta}{1 + \beta}\right)^k \left(\frac{1}{1 + \beta}\right)^r$$

Question 43

Show that a gamma mixture of Poisson random variables gives a negative binomial random variable, and calculate the parameters

Question 44

An insurance company insures a large number of houses. The total number of claims resulting from these policies in a given month is modelled as a negative binomial distribution, and the company estimates that $r = 70$ and $\beta = 0.08$. What is the probability that at least one of these policies leads to a claim in the first month?

Question 45

Show that the limiting case of the negative binomial distribution as $r \to \infty$ and $\beta \to 0$ with $r\beta \to \lambda$, is a Poisson distribution with parameter λ.

6.5 $(a, b, 0)$-class

$(a, b, 0)$-class

A distribution is in the $(a, b, 0)$ class if its probability function satisfies

$$\frac{P(X = k)}{P(X = k - 1)} = a + \frac{b}{k}$$

for all $k \geqslant 1$.

Question 46

Show that the binomial, Poisson and Negative binomial distributions are in the $(a, b, 0)$ class, and calculate the values of a and b in each case.

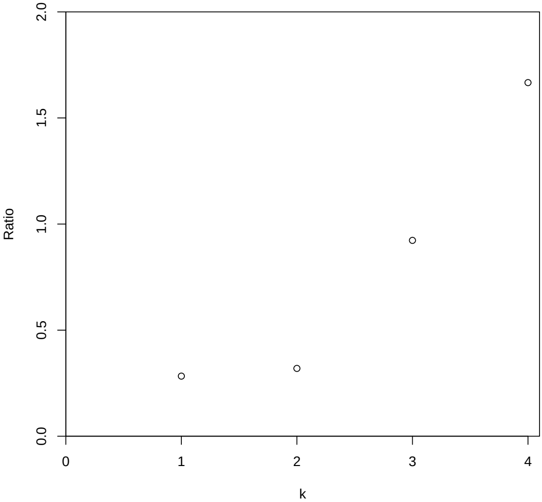
Question 47

The number of claims on each insurance policy over a given time period is observed as follows:

Number of claims	Number of policies
0	861
1	122
2	13
3	3
4	1
5 or more	0

Which distributions from the $(a, b, 0)$-class appear most appropriate for modelling this data?

Plot of $k\frac{n_k}{n_{k-1}}$ against k



$(a, b, 1)$-class

A distribution is in the $(a, b, 1)$ class if its probability function satisfies

$$\frac{P(X = k)}{P(X = k - 1)} = a + \frac{b}{k}$$

for all $k \geqslant 2$.

Question 48

The number of claims resulting from a given insurance policy follows a negative binomial distribution with $r = 3.4$ and $\beta = 0.06$. A company is looking only at the policies that result in at least one claim. If it selects one of these policies, what is the probability that it has resulted in at least 3 claims?

Question 49

The number of claims an individual would make under a certain dental insurance policy in a year follows a negative binomial distribution with $r = 0.8$ and $\beta = 0.7$. However, individuals who would make no claims are less likely to buy the policy. An individual who would make no claims has only a 0.5 probability of buying the policy (but all other individuals buy the policy). What is the distribution of the number of claims made by an individual policyholder.

Question 50

Calculate the probabilites and expected value of an extended truncated negative binomial distribution with $r = -0.6$ and $\beta = 0.8$.

Question 51

Calculate the probabilites and expected value of a logarithmic distribution with $\beta = 0.5$.

Question 52

Consider a compound distribution where the primary distribution is a member of the $(a, b, 0)$ distribution. Find a recurrence relation between the probabilities of the compound distribution.

Question 53

Calculate the probabilities of each of the values 0, 1, and 2 of a compound Poisson-Poisson distribution with parameters λ_1 and λ_2.

Question 54

Show that the binomial-geometric and negative binomial-geometric with r a positive integer, give the same distribution.

Question 55

Show that a compound Poisson-logarithmic distribution gives the same distribution as the negative binomial distribution.

Question 56

Show that a sum of independant compound Poisson random variables is another compound Poisson random variable.

Question 57

(a) Calculate the skewness of a compound Poisson distribution in terms of the first three moments of the secondary distribution.
(b) Use this to calculate the skewness of the Poisson-ETNB distribution.

Question 58

Calculate the probability function of a mixed Poisson distribution with mixing distribution a Pareto distribution with $\alpha = 0.4$ and $\theta = 3$.

Characteristic Functions and Infinite Divisibility

Definition

For a random variable X, the *characteristic function* ϕ_X is given by

$$\phi_X(z) = \mathbb{E}(e^{izx}) = \mathbb{E}(\cos(zX) + i\sin(zX))$$

This is similar to the moment generating function, but it exists for all distributions.

Definition

A distribution with characteristic function $\phi(z)$ is *infinitely divisible* if for any positive integer n, there is another distribution with characteristic function $\phi_n(z)$ such that $(\phi_n(z))^n = \phi(z)$.

This is equivalent to the same statement for the probability generating function or the moment generating function if they exist.

Question 59

Which of the following distributions are infinitely divisible?

a) gamma
b) inverse gamma
c) inverse Gaussian
d) binomial

Question 60

The number of claims arising from a collection of 400 individuals under a group health insurance policy has been modelled as following a compound Poisson-negative binomial distribution with $\lambda = 250$, $r = 2$, $\beta = 0.3$. If the number of individuals insured under the policy increases to 700, what is the new distribution for the number of claims resulting from the policy?

Question 61

The size of loss under a car insurance policy follows a Burr distribution with $\gamma = 0.8$, $\alpha = 1.4$ and $\theta = 3000$. The insurance company is considering adding a deductible of 1000 to the policy.
(a) What is the new distribution of the payment per claim?
(b) If the company introduces a franchise deductible instead, what is the expected value of a claim made under the policy?

Question 62

The severity of a loss on a health insurance policy follows an inverse gamma distribution with $\alpha = 3.2$ and $\theta = 2000$. Calculate the loss elimination ratio of a deductible of \$500.

Question 63

Losses have a Pareto distribution with $\alpha = 2$ and $\theta = k$. There is a deductible of $2k$. Determine the loss elimination ratio before and after inflation of 100%.

Question 64

Losses on a particular insurance policy follow a Weibull distribution with $\theta = 3000$ and $\tau = 3$.
(a) What is the expected loss?
(b) What is the expected loss if a policy limit of $20,000 is imposed on claims.
(c) What is the percentage increase in the expected loss on this policy if there is 20% inflation?

Question 65

Losses follow a Pareto distribution with $\alpha = 2$ and $\theta = 5,000$. An insurance policy pays the following for each loss. There is no insurance payment for the first 1,000. For losses between 1,000 and 6,000, the insurance pays 80%. Losses above 6,000 are paid by the insured until the insured has paid a total payment of 10,000. For any remaining part of the loss, the insurance pays 90%. Determine the expected insurance payment per loss.

Question 66

The loss severity random variable X has an exponential distribution with mean 10,000. Determine the coefficient of variation of the variables Y^P and Y^L, based on $d = 30,000$.

Question 67

Calculate the distribution of the number of claims for a policy with a deductible that reduces the number of claims by a factor of v, if the distribution of the number of losses is:
(a) Binomial with parameters n and p.
(b) Poisson with parameter λ.
(c) Negative binomial with parameters r and β.

Question 68

The frequency distribution for the number of losses when there is no deductible is negative binomial with $r = 3$ and $\beta = 5$. Loss amounts have a Weibull distribution with $\tau = 0.3$ and $\theta = 1,000$. Determine the expected number of payments when a deductible of 200 is applied.

Question 69

The loss amounts under a particular insurance policy follow an inverse Weibull distribution with $\theta = 1000$ and $\tau = 4$. The policy currently has a deductible of \$500, and the number of claims follows a zero-modified ETNB distribution with $r^* = -0.3$, $\beta^* = 7$ and $p_0 = 0.8$. What is the distribution of the number of claims if the deductible is removed?

Question 70

Calculate the VaR and TVaR of the following distribution:

x	$F(x)$	$F(\lceil x \rceil)$
$0 < x < 1$	$0.16x$	0.16
$1 < x < 2$	$0.34x - 0.18$	0.5
$2 < x < 3$	$0.28x - 0.06$	0.78
$3 < x < 4$	$0.19x + 0.21$	0.97
$4 < x < 5$	$0.026x + 0.866$	0.996
$5 < x < 6$	$0.004x + 0.976$	1

(a) At the 90% level
(b) At the 99% level

Question 71

An insurance company models investment risk as following a gamma distribution with $\theta = 2000$ and $\alpha = 4$. Calculate the TVaR of this distribution at the 95% level.

Question 72

An insurance company models the loss on a particular claim as following a mixture distribution: With probability 0.4, the loss follows a Pareto distribution with $\theta = 1000$, $\alpha = 4$. With probability 0.6, the loss follows a Pareto distribution with $\theta = 1000$ and $\alpha = 8$.
(a) Calculate the risk using the standard deviation principle with $a = 3.5$.
(b) Calculate the VaR and TVaR at the 99% level.

Question 73

An insurance company observes the following sample of claims:
2.4, 2.8, 3.5, 3.9, 4.2
It models the distribution of claim sizes using a kernel density model with a uniform kernel with bandwidth 3.
(a) Under this model, what is the median claim size?
(b) What is the median claim size if the company uses a uniform kernel with bandwidth 0.5?

Question 74

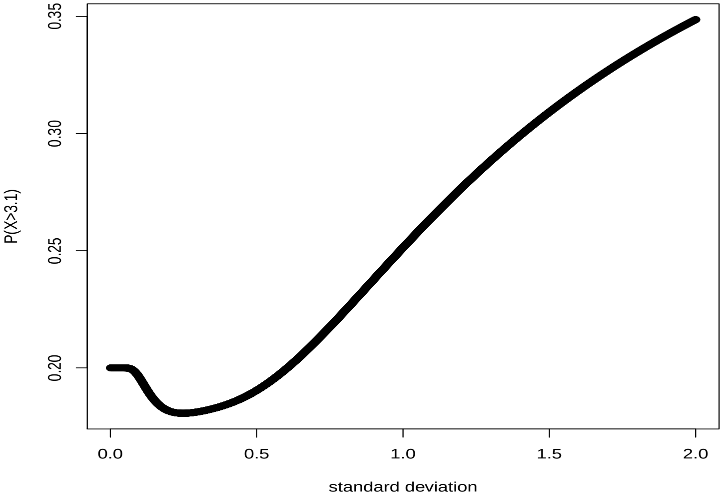
An insurance company observes the following sample of claims:
1.2, 1.4, 2.1, 2.9, 4.3
It models the distribution of claim sizes using a kernel density model with a normal kernel with standard deviation 2. Under this model, what is the probability that a claim is greater than 3.5?

Question 75

An insurance company observes the following sample of claims:
1.8, 2.1, 2.1, 2.4, 3.6
It models the distribution of claim sizes using a kernel density model.
The kernel is a gamma distribution with $\alpha = 3$ and mean equal to the observed point, (so $\theta = \frac{x}{3}$). Calculate the probability under this model that a random claim is greater than 5.

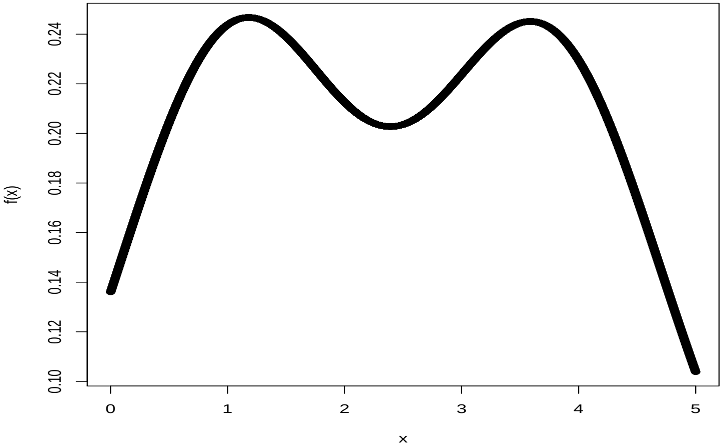
Question 76

An insurance company observes the following sample of claims:
1.4, 1.9, 2.0, 2.8, 3.3
It models the distribution of claim sizes using a kernel density model with Gaussian (normal) kernel. Under this model, what is the probability that a random claim is greater than 3.1 for various values of standard deviation of the kernel distribution?


0.35
0.30
0.25
0.20
P(X>3.1)
0.0
0.5
1.0
1.5
2.0
standard deviation

Question 77

An insurance company observes the following sample of claims:
1.4, 1.5, 1.7, 3.5, 3.7, 3.9
It models the distribution of claim sizes using a kernel density model with Gaussian (normal) kernel with standard deviation 1. Under this model, what is the probability density function for the size of a random claim?



$$f(x) = \frac{1}{6\sqrt{2\pi}} \left(e^{-\frac{(x-1.4)^2}{2}} + e^{-\frac{(x-1.5)^2}{2}} + e^{-\frac{(x-1.7)^2}{2}} + e^{-\frac{(x-3.5)^2}{2}} + e^{-\frac{(x-3.7)^2}{2}} + e^{-\frac{(x-3.9)^2}{2}} \right)$$

Question 78

Each house has a risk factor Θ, which follows a Pareto distribution with $\theta = 1000$ and $\alpha = 2$. Given that a house has risk factor θ, the size of a loss occuring on that home insurance policy follows a Pareto distribution with $\alpha = 3$. What is the probability that a claim on a randomly chosen house exceeds \$1,000.

Question 79

The time until a particular policy results in a claim follows an inverse Weibull distribution with $\tau = 2$ and θ varying between policies. The value of θ for a random policy follows an exponential distribution with mean 4.

(a) What is the probability that a policy produces no claims for 6 years?

(b) Given that a policy has produced no claims for 6 years, what is the probability that it produces no claim the following year?

Question 80

The mortality rate (hazard rate) of an individual aged x is $\lambda e^{0.08x}$, where λ varies between individuals. The value of λ for a random individual at birth follows a gamma distribution with $\theta = 0.00001$ and $\alpha = 3$.

What is the probability that a randomly chosen individual aged 40 survives to age 90?

Question 81

An insurance company divides claims under a certain policy into "small claims" and "large claims", where it defines "small claims" as claims with value under \$2,000, and "large claims" as claims over \$2,000. It models small claims as following a truncated gamma distribution with $\alpha = 3$ and $\theta = 1000$, and models large claims as following a (truncated) Pareto distribution with $\alpha = 4$ and $\theta = 3000$. It wants the overall distribution to have a continuous density function. What proportion of small and large claims must it set in order to ensure this?

Question 82

An insurance company models the cost of a claim for a particular policyholder as following a Pareto distribution with $\alpha = 3$ and θ depending on the policyholder. For a random policyholder, this θ follows a gamma distribution with $\alpha = 2$ and $\theta = 3000$. What is the variance of the cost of a random claim?

Question 83

An insurance company models the cost of a claim as following an inverse gamma distribution with $\alpha = 2$ and θ varying between policies. For a random policy, θ follows a gamma distribution with $\theta = 1000$ and $\alpha = 2$. Calculate the VaR and TVaR of a random policy at the 95% level.

Question 84

The number of individuals making claims in a given year follows a negative binomial distribution with $r = 3$ and $\beta = 1.8$. For individuals who make claims in a given year, the number of claims made follows a truncated negative binomial distribution with $r = 0.3$ and $\beta = 1.6$.
(a) What is the probability that the total number of claims in a given year is 0?
(b) What is the probability that the total number of claims in a given year is 2?

Question 85

The number of incidents in a given year follows a Poisson distribution with $\lambda = 6$. For an incident, the number of claims resulting from that incident follows a negative binomial distribution with $r = 0.3$ and $\beta = 1.6$.
(a) What is the probability that the total number of claims in a given year is 0?
(b) What is the probability that the total number of claims in a given year is 2?

Question 86

The number of claims follows a compound Poisson-Poisson distribution with parameters 4 and 6. Calculate the probability that there are exactly 3 claims.

Question 87

The number of claims for fires follows a compound Poisson-Poisson distribution with parameters 4 and 6. The number of claims for floods follows a compound Poisson-Poisson distribution with parameters 8 and 2. Calculate the probability that there are exactly 2 claims.

Question 88

A group life insurance policy covers 2000 lives in one year. The number of deaths follows a compound negative binomial-Poisson distribution with parameters $r = 2$, $\beta = 1.4$ and $\lambda = 3$. The following year, the number of lives insured has increased to 2600. What is the probability that the number of claims is exactly 3?

Question 89

A group health insurance policy covers 1200 individuals in one year. The number of claims follows a compound Poisson-truncated logarithmic distribution distribution with parameters $\lambda = 3$ and $\beta = 1.4$. The following year, the number of individuals insured has decreased to 900. What is the probability that the number of claims is exactly 3?