ACSC/STAT 3703, Actuarial Models I (Further Probability with Applications to Actuarial Science) WINTER 2015

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Sample Midterm Examination

This Sample examination has more questions than the actual midterm, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. The random variable X has density function given by

$$f(x) = \frac{15}{4}x(1-x)^2(2-x), 0 \leqslant x \leqslant 2$$

   (a) calculate the hazard rate of X. [10 mins]

   (b) Calculate the kurtosis of X [10 mins]

2. Losses follow a Pareto distribution with $\alpha = 3$. How large can θ be if the insurance company wants to limit its Value at Risk at the 95% level to \$15,000? [10 mins]

3. Calculate the moment generating function of a sum of 5 independent beta random variables with parameters 3 and 2. [10 mins]

4. Which distribution has a heavier tail: a gamma distribution with $\alpha = 4$ and $\theta = 400$, or a Weibull distribution with $\tau = 4$ and $\theta = 400$? [Use any reasonable method for comparing tail-weight.] [5 mins.]

5. Recall that desirable coherence properties for measures of risk are:

   - Subadditivity
   - Monotonicity
   - Positive homogeneity
   - Translation invariance

   Which properties are satisfied by the risk measure given by the measure $r(X) = \mu + \pi_{0.9}$ (the mean plus the 90th percentile)? [15 mins]

6. Calculate the TVaR of a gamma distribution with $\alpha = 3$ and $\theta = 2000$ at the 0.99 level. [The VaR at the 0.99 level is 16,811.894] [15 mins]

7. Claims follow a Pareto distribution with $\alpha = 4$. There is a policy limit which is currently exceeded by 0.16% of claims. (That is $l = \pi_{0.9984}$.) There is uniform inflation of 8% per year on claim amounts. What proportion of claims will exceed the policy limit in 4 years time? That is, what percentile is l in 4 years' time? [The policy limit does not change in these 4 years.] [10 mins]

8. An insurance company deals with three types of claim:

   - 10% of claims are for fire damage. These claims follow a Pareto distribution with $\alpha = 3$ and $\theta = 300,000$.
   - 60% of claims are for weather damage. These claims follow a Weibull distribution with $\tau = \frac{1}{3}$ and $\theta = 10,800$.
   - The remaining 30% of claims are for break-ins. These claims follow a Weibull distribution with $\tau = 3$ and $\theta = 1000$.

   Calculate the probability that a randomly chosen claim exceeds $400,000. [15 mins]

9. You observe the following sample of insurance losses:

   1.6 3.6 3.8 4.2 5.6

   Using a Kernel density model with Gaussian (normal) kernel with standard deviation 1.2, estimate the probability that a loss exceeds 5.5. [10 mins]

10. You observe the following sample of insurance losses:

    1.6 3.6 3.8 4.2 5.6

    Using a Kernel density model with triangular kernel with bandwidth 2, estimate the probability that a loss exceeds 5.5. A triangular kernel with bandwidth b centred at x_0 is given by the density function

    $$f(x) = \begin{cases} \frac{x+b-x_0}{b^2} & \text{if } x_0 - b < x < x_0 \\ \frac{x_0+b-x}{b^2} & \text{if } x_0 < x < x_0 + b \\ 0 & \text{otherwise} \end{cases}$$

    [10 mins]

11. An insurance company models its investment gains over a period of t years as e^X where X follows a gamma distribution with parameters $\alpha = 2t$ and $\theta = 0.05$. Calculate the density function for its investment gains over a 4-year period. [10 mins]

12. Given $\Theta = \theta$, the lifetime of a computer follows an inverse exponential distribution with parameter θ. Θ follows a gamma distribution with $\alpha = 2$, and $\theta = 3$. What is the distribution of the lifetime of a randomly chosen computer? [10 mins]

13. The mortality rate of a man aged x is modelled as being $\lambda e^{0.1x}$, where λ follows a gamma distribution with $\alpha = 3$ and $\theta = 0.001$. Calculate the probability of a man aged 40 surviving to age 90. [10 mins]

14. Recall that the limit of a transformed beta distribution as $\tau \to \infty$, $\theta \to 0$ and $\theta\tau^{\frac{1}{\gamma}} \to \xi$ is an inverse gamma with $\theta = \xi$ and $\alpha = \alpha$. What is the limit of an inverse Pareto distribution as $\tau \to \infty$ and $\theta \to 0$ with $\tau\theta = \xi$. [5 mins]

15. Let X have density function given by

$$f(x) = \left(\frac{\theta}{1+\theta}\right)^x \log\left(\frac{1+\theta}{\theta}\right)$$

for $0 < x$.

(a) Show that the distribution of X is from the linear exponential family, and calculate the functions $p(x)$, $q(\theta)$, and $r(\theta)$. [5 mins.]

(b) Calculate the variance of X as a function of θ. [5 mins.]

16. The number of claims experienced by an insurance company in a given year follows a Poisson distribution with mean 30. Of these claims, 10% are for fires and 15% are for floods. What is the probability that in a given year the company experiences exactly 2 claims for fires and at most 2 claims for floods? [10 mins]

17. For a driver with safety rating $\Theta = \theta$, the number of claims made in a year follows a Poisson distribution with parameter θ. For a random driver, Θ follows a gamma distribution with parameter $\alpha = 4$ and $\theta = 0.2$. What is the probability that a randomly chosen driver makes no claims in a given year? [10 mins]

18. An insurance company models the number of claims received with a distribution from the $(a, b, 1)$-class. It calculates that the probability of receiving exactly 5 claims is 0.1; the probability of receiving exactly 6 claims is 0.04; the probability of receiving exactly 8 claims is 0.0027. What is the modified probability of receiving no claims? [15 mins]

19. An insurance company observes the following claim numbers on a group insurance policy:

Number of Claims	Frequency
0	12,345
1	4,521
2	874
3	130
4	17
5	2
6 or more	0

By calculating $k\frac{p_k}{p_{k-1}}$, decide which distributions from the $(a, b, 0)$-class are most appropriate. [10 mins]

20. An insurance company models loss frequency for an individual as following a zero-modified logarithmic distribution with $p_0 = 0.8$ and $\beta = 3$. What is the probability that this individual experiences at least 3 losses? [10 mins]

21. Losses follow a compond Poisson-Poisson distribution with parameters 2 and 4. Calculate the probability that there are more than 2 losses. [10 mins]

22. The number of claims on policies from one group with 200 members follows a compound Poisson-Geometric distribution with parameters $\lambda = 3$ and $\beta = 2$. The number of claims on policies from another group with 250 members follows a compound Poisson-Poisson distribution with parameters 2 and 1. Next year, the first group is increasing to 400 members and the second group is increasing to 400 members. Calculate the probability that there are at least 3 claims next year. [15 mins]

23. The number of claims follows a mixture distribution. Given $\Theta = \theta$, the number of claims follows a negative binomial distribution with $r = 4$ and $\beta = \frac{\theta}{4}$. Θ follows a Pareto distribution with $\theta = 4$ and $\alpha = 3$. What is the probability that the number of claims is exactly 3? [10 mins]