ACSC/STAT 3720, Life Contingencies I
Winter 2017
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Homework Sheet 2
Model Solutions

Basic Questions

1. *Using the select lifetable in Table 1, calculate:*

   (a) the probability that an individual aged 39 who was select 1 year ago dies within the next 5 years.

   This probability is given by $1 - \frac{l_{[41]+3}}{l_{[38]+1}} = 1 - \frac{9936.94}{9961.14} = 0.0024294$.

   (b) the probability that an individual aged 44 who was select 6 years ago dies within the next 8 years.

   This individual is in the ultimate part of the model, so the probability is $1 - \frac{l_{[49]+3}}{l_{[41]+3}} = 1 - \frac{9865.30}{9936.94} = 0.007209463$.

   (c) the probability that an individual aged 33 who is select survives to age 80.

   This probability is given by $\frac{l_{[77]+3}}{l_{[33]}} = \frac{8423.00}{9981.07} = 0.8438975$.

   (d) ${}_{4|7}q_{[35]+1}$

   This probability is given by $\frac{l_{[37]+3}-l_{[44]+3}}{l_{[35]+1}} = \frac{9958.44-9915.52}{9972.79} = 0.00430371$.

2. *An individual's mortality follows the select lifetable in Table 1. You are given that for this lifetable, the expected curtate future lifetime for an individual at age 65 in the ultimate part of the model is 29.04. Calculate the expected curtate future lifetime for*

   (a) a select individual aged 58

   The expected curtate lifetime for a select individual aged 58 is given by

$$\begin{aligned}
&{}_1p_{[58]} + {}_2p_{[58]} + {}_3p_{[58]} + {}_4p_{[58]} + {}_5p_{[58]} + {}_6p_{[58]} + {}_7p_{[58]} + {}_8p_{[58]}(1+29.04)\\
=&\frac{1}{9751.79}\left(9737.56 + 9719.69 + 9697.28 + 9669.17 + 9638.51 + 9605.07 + 9568.61 + 9528.85 \times 30.04\right)\\
=&36.28898
\end{aligned}$$

   (b) an individual aged 60 who was select 2 years ago.

   This is given by

$$\begin{aligned}
&{}_1p_{[58]+2} + {}_2p_{[58]+2} + {}_3p_{[58]+2} + {}_4p_{[58]+2} + {}_5p_{[58]+2} + {}_6p_{[58]+2}(1+29.04)\\
=&\frac{1}{9719.69}\left(9697.28 + 9669.17 + 9638.51 + 9605.07 + 9568.61 + 9528.85 \times 30.04\right)\\
=&34.40699
\end{aligned}$$

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00

3. *Using the ultimate mortality model*

$$\mu_x = 0.000075 + 0.0000146 \times 1.119^x$$

and select mortality

$$\mu_{[x]+s} = 0.9^{3-s}\mu_{x+s}$$

for the selection period of 3 years, calculate a select life table between ages 41 and 45 using radix 10,000. [You may use the approximation $q_{[x]+t} = \mu_{[x+0.5]+t}$*.]*

We start by forming the ultimate part of the lifetable.

x	l_x	d_x
41	10000.00	16.27
42	9983.73	18.08
43	9965.65	20.11
44	9945.54	22.37
45	9923.17	24.89
46	9898.28	27.69
47	9870.59	30.81
48	9839.78	34.28

This gives us the final column of the table. We then compute the select part of the lifetable by calculating $q_{[x]+s} = 0.9^{3-s}q_{x+s}$. For example,

$$q_{[41]+2} = 0.9q_{43} = 0.9 \times \frac{20.11}{9965.65} = 0.001816138$$

From this, we get $l_{[41]+2} = \frac{9945.54}{1-0.001816138} = 9963.63$. Completing the other entries in a similar way gives the following table:

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
41	9990.12	9978.27	9963.63	9945.54
42	9972.75	9959.58	9943.30	9923.17
43	9953.42	9938.78	9920.68	9898.28
44	9931.94	9915.65	9895.51	9870.59
45	9908.03	9889.91	9867.50	9839.78

4. *The lifetable in Table 1 applied 5 years ago. The following is an excerpt from the ultimate part of an updated lifetable:*

x	l_x	d_x
40	*10000.00*	*4.16*
41	*9995.84*	*4.54*
42	*9991.30*	*4.96*
43	*9986.34*	*5.43*
44	*9980.92*	*5.94*
45	*9974.97*	*6.51*

Calculate the reduction factor used for each age between 40 and 45.

We calculate the original q_x and the updated q_x. The reduction factor is the ratio to the power $\frac{1}{5}$.

x	old q_x	new q_x	Ratio of new q_x/old q_x	Reduction factor
40	0.0004769823	0.0004160000	0.8721497	0.9730120
41	0.0005163914	0.0004541889	0.8795440	0.9746563
42	0.0005598806	0.0004964319	0.8866746	0.9762316
43	0.0006074638	0.0005437428	0.8951032	0.9780805
44	0.0006601630	0.0005951355	0.9014979	0.9794741
45	0.0007169917	0.0006526335	0.9102386	0.9813661

Standard Questions

5. *An insurance company determines that individuals aged 30 who do not pass the underwriting process used for policies following Table 1 have probability 0.096 of dying within the next five years. What is the probability that a select individual aged 26 would pass the underwriting process in four years' time (at age 30). [Either an individual passes the underwriting process, in which case their mortality is that of a select life at their age, or they do not pass the underwriting process.]*

Let s denote the probability of passing the underwriting process at age 30. The probability of surviving to age 35 is then given by

$$ {}_9p_{[26]} = s\,{}_5p_{[30]} + ({}_4p_{[26]} - s)(1 - 0.096) $$

The first term is the probability of being select in 4 years time and surviving to age 35. The second is the probability of not being select and surviving to age 35. We can substitute the numbers from the lifetable, to get

$$ \begin{aligned} \frac{9977.37}{9997.00} &= \frac{9977.37}{9988.81}s + 0.904\left(\frac{9990.52}{9997.00} - s\right) \\ \left(\frac{9977.37}{9988.81} - 0.904\right)s &= \frac{9977.37}{9997.00} - 0.904 \times \frac{9990.52}{9997.00} \\ s &= \frac{9988.81(9977.37 - 0.904 \times 9990.52)}{9997.00(9977.37 - 9988.81 \times 0.904)} \qquad = 0.9975506 \end{aligned} $$

6. *An insurance company has used Makeham's formula with a constant factor to discount for selected lives — that is $\mu_{[x]+s} = D^{3-s}\mu_{x+s}$ to construct a lifetable for female smokers. The lifetable is given below.*

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
40	*9968.27*	*9946.76*	*9917.80*	*9878.81*
41	*9928.58*	*9904.00*	*9870.91*	*9826.37*
42	*9883.21*	*9855.14*	*9817.34*	*9766.48*
43	*9831.37*	*9799.32*	*9756.17*	*9698.12*
44	*9772.16*	*9735.58*	*9686.35*	*9620.16*

Find the parameters used in the model to produce the table. [It was constructed using the approximation $q_{[x]+s} = \mu_{[x+0.5]+s}$.]

We are able to calculate different values of $q_{[x]+s}$ as ratios of entries in this table. For example, we have $q_{[40]+3} = 1 - \frac{9826.37}{9878.81} = 0.005308332$ and $q_{[41]+2} = 1 - \frac{9826.37}{9870.91} = 0.004512249$. This gives us $D = \frac{0.004512249}{0.005308332} = 0.8500314$.

For the other parameters, we can use the ultimate part of the lifetable:

$$\begin{aligned}
q_{[41]+3} &= 1 - \frac{9766.48}{9826.37} = 0.006094824 \\
q_{[42]+3} &= 1 - \frac{9698.12}{9766.48} = 0.006999451 \\
q_{[43]+3} &= 1 - \frac{9620.16}{9698.12} = 0.008038671
\end{aligned}$$

Substituting the formula $q_{[x]+3} = A + BC^{x+3.5}$, we get

$$\begin{aligned}
A + BC^{43.5} &= 0.005308332 \\
A + BC^{44.5} &= 0.006094824 \\
A + BC^{45.5} &= 0.006999451 \\
A + BC^{46.5} &= 0.008038671
\end{aligned}$$

Subtracting the first equation from the second and the third from the fourth gives

$$\begin{aligned}
&= 0.005308332 \\
B\left(C^{44.5} - C^{43.5}\right) &= 0.000786492 \\
&= 0.005308332 \\
B\left(C^{46.5} - C^{45.5}\right) &= 0.00103922 \\
C^2 &= \frac{0.00103922}{0.000786492} = 1.321336 \\
C &= 1.149494
\end{aligned}$$

Substituting this into the earlier equation gives

$$B = \frac{0.000786492}{1.149494^{44.5} - 1.149494^{43.5}} = 0.0000122752$$

This gives

$$A = 0.005308332 - 0.0000122752 \times 1.149494^{43.5} = 0.0000473065$$

[There are different ways to use the equations to estimate these parameters, so your answers may vary slightly. The actual values used were $A = 0.000066, B = 0.000012, C = 1.15, D = 0.85$. We see that the rounding errors introduce a small change in C, which results in larger changes in B and significantly larger changes in A. This could be slightly improved by comparing mortalities at ages that are further apart.]