ACSC/STAT 3720, Life Contingencies I
Winter 2017
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Homework Sheet 7
Model Solutions

Basic Questions

1. *An insurance company sells 600 whole life insurance policies with annual net premiums to lives aged 38. The death benefit on these policies is $400,000. The interest rate is* $i = 0.06$. *In the first year of the policies:*

   - *No policyholders die.*
   - *The company earns interest* $i = 0.05$.

   The company still uses $i = 0.06$ *as its basis for calculating the policy values. What is the company's annual profit on these policies? [Using the lifetable in Table 1, we have* $A_{38} = 0.0580484$ *and* $A_{39} = 0.0611489$.*]*

   We calculate

$$\ddot{a}_{38} = \frac{1.06}{0.06}(1 - A_{38}) = 16.64114$$
$$\ddot{a}_{39} = \frac{1.06}{0.06}(1 - A_{39}) = 16.5863716.64114$$

   So the net premium is $\frac{400000\times0.0580484}{16.64114} = \$1395.30$ and the policy value after 1 year is $400000\times 0.0611489 - 1395.30 \times 16.58637 = \$1,316.60$.

   The insurance company therefore collects $600 \times 1395.30 = \$837,180$ in premiums, and earns $837180\times0.05 = \$41,859$ in interest, so at the end of the year, it has $879,039 in cash. It needs $600 \times 1316.60 = \$789,960$ in reserves for the outstanding policies. The profit is therefore $879039 - 789960 = \$89,079$.

2. *An insurance company sells 700 25-year term insurance policies to lives aged 45. The death benefit of these policies is $800,000. The interest rate is* $i = 0.045$ *and net premiums are payable annually in advance. At this interest rate,* $A_{45} = 0.142031$ *and* $A_{70} = 0.359627$. *In the first two years of the policy:*

   - *two policyholders die in the first year of the policy.*
   - *The company earns interest* $i = 0.06$ *in the first year of the policy, and* $i = 0.05$ *in the second year.*

   Calculate the asset share of the remaining policies after the second year.

   We calculate $A^{1}_{45:\overline{10|}} = A_{45} - {}_{25}p_{45}(1.045)^{-25}A_{70} = 0.142031 - 0.359627 \times \frac{9330.85}{9930.38}(1.045)^{-25} = 0.0295963$ and

$$\ddot{a}_{45:\overline{10|}} = \frac{1.045}{0.045}\left(1 - 0.142031 - (1 - 0.359627) \times \frac{9823.089330.85}{9930.38}(1.045)^{-25}\right) = 15.27468$$

This means that the annual premium is $\frac{800000\times 0.0295963}{15.27468} = \$1550.08$.

The accumulated value of the premiums at the end of the second year is $805.221550.08(700(1.06)(1.05)+698(1.05)) = \$2,343,720.96$. The accumulated value of benefits paid is $800000 \times 2 \times 1.05 = \$1,680,000$. The remaining assets are therefore $2,343,720.96 - 1680000 = \$663,720.96$. The asset share is therefore $\frac{663720.96}{698} = \$950.89$.

3. *A life aged 37 purchases a whole-life insurance policy with a death benefit of \$600,000. The interest rate is $i = 0.05$. From the lifetable in Table 1, we have $A_{37} = 0.0827855$ and $A_{44} = 0.112997$. Using Woolhouse's formula, this results in a monthly premium of \$225.15.*

   (a) calculate the policy value after 6 years and 9 months. [You may use the UDD assumption for the distribution of deaths in Year 7, but use Woolhouse's formula to calculate $\ddot{a}_{44}^{(12)}$.]

   We have that $\ddot{a}_{44} = \frac{1.05}{0.05}(1 - 0.112997) = 18.62706$. Using Woolhouse's formula, with

$$\mu_{44} \approx \frac{1}{2}(q_{43} + q_{44}) = \frac{1}{2}\left(\frac{6.04}{9942.98} + \frac{6.56}{9936.94}\right) = 0.0006338134$$

   we get

$$\ddot{a}_{44}^{(12)} = \ddot{a}_{44} - \frac{11}{24} - \frac{143}{1728}(\log(1.05) + 0.0006338134) = 18.16464$$

   We have that $q_{43} = \frac{6.04}{9942.98}$, so under the UDD assumption, ${}_{\frac{3}{4}}q_{43} = \frac{3}{4} \times \frac{6.04}{9942.98} = \frac{4.53}{9942.98}$ and ${}_{\frac{3}{4}}p_{43} = \frac{9438.45}{9942.98}$. The probability of dying in each remaining month of the year is then $\frac{1}{12} \times \frac{6.04}{9438.45} = 0.00005332797$. We then use the recurrence

$$\ddot{a}_{43\frac{3}{4}}^{(12)} = \frac{1}{12}\left(1 + 0.9999467(1.05)^{-\frac{1}{12}} + 0.9998933(1.05)^{-\frac{2}{12}}\right) + 0.9998400(1.05)^{-\frac{3}{12}} \times 18.16464 = 18.10719$$

   This gives $A_{43\frac{3}{4}}^{(12)} = 1 - d^{(12)} \times 18.10719 = 1 - 12 \times \frac{1.05^{\frac{1}{12}} - 1}{1.05^{\frac{1}{12}}} \times 18.10719 = 0.1183408$, and therefore, the policy value is $0.1183408 \times 600000 - 18.10719 \times 225.15 \times 12 = \$22,082.47$.

   (b) calculate the policy value after 6 years 8.2 months.

   Using the UDD assumption, we calculate

$${}_{\frac{0.8}{12}}q_{43+\frac{8.2}{12}} = \frac{\frac{0.8}{12} \times 6.04}{\frac{8.2}{12} \times 9936.94 + \frac{3.8}{12} \times 9942.98} = 0.0000405144$$

   so the policy value after 6 years 8.2 months is

$$(0.0000405144 \times 600000 + (1 - 0.0000405144) \times 22082.47)(1.05)^{-\frac{0.8}{12}} = \$22,034.10$$

Standard Questions

4. *An insurance company wants to design a 10-year term policy with continuous premiums so that the policy value is given by ${}_tV = 150t(t-10)(t-15)$. The death benefits at time t are $100000(3 + 0.1t)$. The policy is sold to a life aged 36, with mortality given by $\mu_x = 0.0000045(1.12)^x$. Calculate the premiums as a function of time if force of interest is $\delta = 0.051$.*

Thiel's differential equation gives

$$\frac{d}{dt}{}_tV = \delta_t V + P_t - \mu_{x+t}(S_t -_t V)$$

$$\begin{aligned}
P_t &= \frac{d}{dt}{}_tV - \delta_t V + \mu_{x+t}(S_t -_t V) \\
&= 150(3t^2 - 50t + 150) - 0.051 \times 150t(t-10)(t-15) + \mu_{36+t}(100000(3+0.1t) - 150t(t-10)(t-15)) \\
&= (22500 - 8647.5t + 641.25t^2 - 7.65t^3) + 0.0000045(1.12)^{36+t}(300000 - 12500t + 3750t^2 - 150t^3) \\
&= 22500 - 8647.5t + 641.25t^2 - 7.65t^3 + 0.0002661101(1.12)^t(300000 - 12500t + 3750t^2 - 150t^3)
\end{aligned}$$

5. *An insurance company is valuing its policies. It finds that the total value of a large group of 200 policies was \$16,000,000. The total annual premium for all these policies is \$1,800,000. The interest rate is $i = 0.06$. All of the policies have a mortality rate $q_x = 0.07$. There are three types of policy in the portfolio — 120 of the policies are type 1 and have death benefit \$900,000; 40 are type 2 and have death benefit \$1,500,000; and the remaining 40 are type 3 and have death benefit \$1,300,000. There are no expenses associated with the policies, and during the following year 6 type 1 policyholders die, 2 type 2 policyholders die and 2 type 3 policyholders die. What is the total value of all the remaining policies the following year?*

   Let the values of the three types of policies at the start of the year be u_1, u_2 and u_3. Let the values of each type of policy at the end of the year be v_1, v_2 and v_3 respectively. Let the premiums at the start of the year for the three types of policies be p_1, p_2, and p_3 respectively. From our recurrence, we get

$$u_1 = (0.93v_1 + 0.07 \times 900000)1.06^{-1} - p_1 \quad (1)$$
$$u_2 = (0.93v_2 + 0.07 \times 1500000)1.06^{-1} - p_2 \quad (2)$$
$$u_3 = (0.93v_3 + 0.07 \times 1300000)1.06^{-1} - p_3 \quad (3)$$

   We are given

$$\begin{aligned}
120u_1 + 40u_2 + 40u_3 &= 16000000 \\
120p_1 + 40p_2 + 40p_3 &= 1800000
\end{aligned}$$

   and we are asked to calculate $114v_1 + 38v_2 + 38v_3 = 0.95(120v_1 + 40v_2 + 40v_3)$. Adding 120 times Equation (1), 40 times Equation (2) and 40 times Equation (3) gives:

$$\begin{aligned}
120u_1 + 40u_2 + 40u_3 &= (0.93(120v_1 + 40v_2 + 40v_3) + 15400000)(1.06)^{-1} - (120p_1 + 40p_2 + 40p_3) \\
16000000 &= (0.93(120v_1 + 40v_2 + 40v_3) + 15400000)(1.06)^{-1} - 1800000
\end{aligned}$$

$$\begin{aligned}
(0.93(120v_1 + 40v_2 + 40v_3) + 15400000) &= (1.06)(16000000 + 1800000) \\
0.93(120v_1 + 40v_2 + 40v_3) &= 18868000 - 15400000 \\
&= 3468000
\end{aligned}$$

   The total policy value of the remaining policies at the end of the year is therefore $\frac{3468000 \times 0.95}{0.93} =$ \$354, 2580.65.

Table 1: Select lifetable to be used for questions on this assignment

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
25	9998.75	9997.65	9996.30	9994.66
26	9997.00	9995.83	9994.40	9992.66
27	9995.14	9993.90	9992.38	9990.52
28	9993.16	9991.84	9990.22	9988.24
29	9991.05	9989.65	9987.92	9985.80
30	9988.81	9987.30	9985.46	9983.18
31	9986.40	9984.80	9982.82	9980.38
32	9983.83	9982.11	9979.99	9977.37
33	9981.07	9979.23	9976.95	9974.13
34	9978.11	9976.13	9973.68	9970.64
35	9974.93	9972.79	9970.16	9966.88
36	9971.50	9969.20	9966.36	9962.82
37	9967.80	9965.33	9962.25	9958.44
38	9963.81	9961.14	9957.82	9953.69
39	9959.50	9956.61	9953.02	9948.55
40	9954.84	9951.71	9947.82	9942.98
41	9949.79	9946.41	9942.19	9936.94
42	9944.32	9940.66	9936.08	9930.38
43	9938.39	9934.41	9929.45	9923.26
44	9931.96	9927.64	9922.25	9915.52
45	9924.97	9920.28	9914.42	9907.10
46	9917.37	9912.28	9905.91	9897.94
47	9909.11	9903.58	9896.65	9887.98
48	9900.13	9894.11	9886.57	9877.13
49	9890.36	9883.80	9875.59	9865.30
50	9879.71	9872.57	9863.63	9852.42
51	9868.12	9860.34	9850.59	9838.38
52	9855.48	9847.01	9836.39	9823.08
53	9841.72	9832.48	9820.90	9806.39
54	9826.71	9816.64	9804.02	9788.18
55	9810.34	9799.37	9785.60	9768.33
56	9792.49	9780.52	9765.51	9746.67
57	9773.03	9759.97	9743.60	9723.05
58	9751.79	9737.56	9719.69	9697.28
59	9728.63	9713.10	9693.62	9669.17
60	9703.36	9686.43	9665.17	9638.51
61	9675.80	9657.33	9634.15	9605.07
62	9645.73	9625.59	9600.31	9568.61
63	9612.94	9590.98	9563.42	9528.85
64	9577.18	9553.24	9523.19	9485.52
65	9538.19	9512.09	9479.35	9438.30
66	9495.69	9467.25	9431.58	9386.86
67	9449.37	9418.39	9379.54	9330.85
68	9398.90	9365.17	9322.87	9269.88
69	9343.95	9307.23	9261.20	9203.55
70	9284.12	9244.18	9194.11	9131.43
71	9219.03	9175.59	9121.17	9053.07
72	9148.24	9101.03	9041.91	8967.97
73	9071.30	9020.03	8955.85	8875.63

x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	$l_{[x]+3}$
74	8987.73	8932.10	8862.49	8775.52
75	8897.04	8836.71	8761.27	8667.10
76	8798.69	8733.34	8651.66	8549.78
77	8692.13	8621.41	8533.09	8423.00
78	8576.81	8500.36	8404.95	8286.16
79	8452.13	8369.60	8266.68	8138.66
80	8317.52	8228.53	8117.67	7979.93
81	8172.36	8076.57	7957.35	7809.41
82	8016.08	7913.13	7785.15	7626.56
83	7848.11	7737.67	7600.54	7430.89
84	7667.89	7549.66	7403.05	7221.99
85	7474.92	7348.64	7192.27	6999.51
86	7268.77	7134.21	6967.86	6763.22
87	7049.07	6906.07	6729.62	6513.04
88	6815.55	6664.05	6477.46	6249.02
89	6568.09	6408.10	6211.48	5971.42
90	6306.70	6138.35	5931.96	5680.73
91	6031.59	5855.15	5639.41	5377.67
92	5743.19	5559.08	5334.61	5063.27
93	5442.15	5250.97	5018.61	4738.86
94	5129.44	4931.97	4692.79	4406.12
95	4806.33	4603.54	4358.89	4067.08
96	4474.39	4267.51	4018.96	3724.10
97	4135.60	3926.04	3675.44	3379.91
98	3792.25	3581.66	3331.11	3037.57
99	3447.02	3237.23	2989.05	2700.39
100	3102.90	2895.94	2652.63	2371.88
101	2763.19	2561.21	2325.37	2055.64
102	2431.39	2236.61	2010.90	1755.27
103	2111.15	1925.80	1712.81	1474.18
104	1806.12	1632.34	1434.48	1215.44
105	1519.82	1359.55	1178.94	981.65
106	1255.46	1110.36	948.70	774.71
107	1015.81	887.14	745.58	595.71
108	802.96	691.49	570.56	444.87
109	618.23	524.17	423.71	321.41
110	462.04	385.00	304.13	223.65
111	333.80	272.80	210.00	149.10
112	231.99	185.53	138.71	94.62
113	154.19	120.34	87.07	56.74
114	97.30	73.90	51.50	31.84
115	57.78	42.55	28.41	16.52
116	31.92	22.69	14.43	7.81
117	16.15	11.04	6.63	3.30
118	7.34	4.79	2.69	1.21
119	2.90	1.79	0.93	0.37
120	0.95	0.55	0.26	0.09
121	0.23	0.13	0.05	0.01
122	0.03	0.02	0.01	0.00