ACSC/STAT 4703, Actuarial Models II
Fall 2018
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Homework Sheet 2
Due: Friday 5th October: 11:30 PM

Basic Questions

1. An insurance company has the following portfolio of workers compensation insurance policies:

Type of worker	Number	Probability of claim	mean claim	standard deviation
Engineer	1300	0.015	\$46,000	\$88,000
Salesperson	1100	0.005	\$29,000	\$32,000
Manager	150	0.001	\$20,000	\$28,000

Calculate the cost of reinsuring losses above \$3,000,000, if the loading on the reinsurance premium is one standard deviation above the expected claim payment on the reinsurance policy using a Gamma approximation for the aggregate losses on this portfolio.

2. An insurance company is modelling claim data as following a Weibull distribution with $\tau = 0.7$. It collects the following sample of claims:

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16.3  22.3  37.5  38.6  68.6  69.7  79.1  85.8  142.9  158.5
175.2  176.1  205.1  265.5  266.9  287.3  299.8  354.2  357.4
365.9  391.9  407.9  613.4  692.4  745.2  771.3  845.9  1780.3
1795.5  1994.7
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The MLE for θ is 380.1094. Graphically compare this empirical distribution with the best fitting Weibull distribution with $\tau = 0.7$. Include the following plots:

(a) Comparisons of $F(x)$ and $F^*(x)$

(b) Comparisons of $f(x)$ and $f^*(x)$

(c) A plot of $D(x)$ against x.

(d) A p-p plot of $F(x)$ against $F^*(x)$.

3. For the data in Question 2, calculate the following test statistics for the goodness of fit of the Weibull distribution with $\tau = 0.7$ and $\theta = 380.1094$:

(a) The Kolmogorov-Smirnov test.

(b) The Anderson-Darling test.

(c) The chi-square test, dividing into the intervals 0–200, 200–400, and more than 400.

4. For the data in Question 2, perform a likelihood ratio test to determine whether a Weibull distribution with fixed $\tau = 0.7$, or a Weibull distribution with τ freely estimated is a better fit for the data. [The MLE for the general Weibull distribution is $\tau = 0.3125$ and $\theta = 295.7674$.]

5. For the data in Question 2, use AIC and BIC to choose between a Weibull distribution with $\tau = 0.7$ and a Pareto distribution for the data. [The MLE for the Pareto distribution is $\alpha = 4.8761$ and $\theta = 1760.6118$.]

Standard Questions

6. An insurance company insures three types of properties and has the following estimates:

Property type	Probability of claim	mean claim	standard deviation
Residential (House)	0.004	\$8,600	\$25,800
Residential (Apartment)	0.009	\$2,300	\$6,900
Commercial	0.02	\$3,600	\$12,400

The insurance company estimates the mean μ and standard deviation σ for the aggregate loss distribution, and buys stop-loss insurance for losses above \$200,000. One reinsurer models aggregate losses as following a Pareto distribution and sets its premium as 110% of the expected claims on the stop-loss policy. Another reinsurer models aggregate losses as following a Gamma distribution, and sets its premium at 200% of the expected claims. The portfolio includes 2,243 houses and 1,832 apartments. How many commercial properties would it need to include for the two reinsurance companies to charge the same premium on the stop-loss insurance?

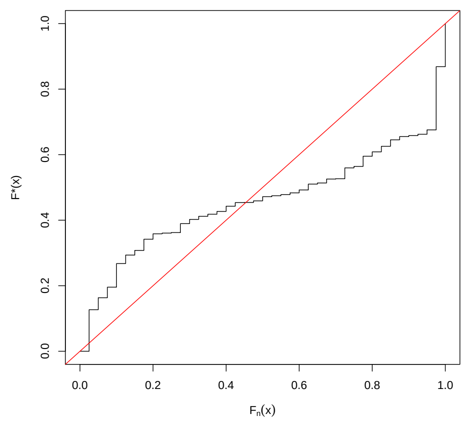
(i) 640

(ii) 1,209

(iii) 1,853

(iv) 2,177

7. An insurance company collects a sample of 40 past claims, and attempts to fit a distribution to the claims. Based on experience with other claims, the company believes that a Pareto distribution with $\alpha = 3$ and $\theta = 1,200$ may be appropriate to model these claims. It constructs the following p-p plot to compare the sample to this distribution:



(a) How many of the points in their sample were less than 168?

(b) Which of the following statements best describes the fit of the Pareto distribution to the data:

(i) The Pareto distribution assigns too much probability to high values and too little probability to low values.

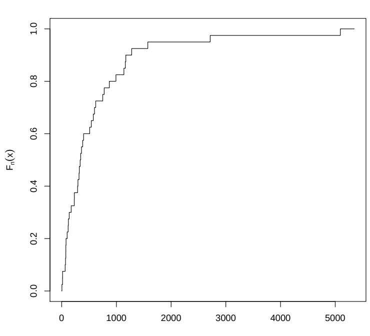
(ii) The Pareto distribution assigns too much probability to low values and too little probability to high values.

(iii) The Pareto distribution assigns too much probability to tail values and too little probability to central values.

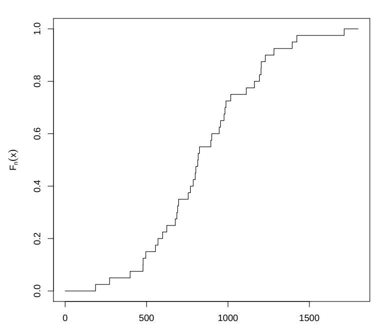
(iv) The Pareto distribution assigns too much probability to central values and too little probability to tail values.

(c) Which of the following plots shows the empirical distribution function? Justify your answer.

(i)



(ii)



(iii)