ACSC/STAT 4703, Actuarial Models II

Fall 2018

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Homework Sheet 3

Model Solutions

Basic Questions

1. *A homeowner's house is valued at \$520,000, but is insured at \$270,000. The insurer requires 70% coverage for full insurance. The home sustains \$8,400 from flooding. The policy has a deductible of \$5,000, which decreases linearly to zero when the total cost of the loss is \$10,000. How much does the insurance company reimburse?*

   The proportion of coverage is $\frac{270000}{0.7\times 520000} = 0.741758241758$. The deductible is $5000\frac{10000-8400}{10000-5000} = \$1,600$. The insurance therefore pays $(8400-1600)\times 0.741758241758 = \$5,043.96$.

2. *An insurance company has three types of coverages for businesses with different expected loss ratios, and has the following data on recent claims:*

Policy Type	*Policy Year*	*Earned Premiums*	*Expected Loss Ratio*	*Losses paid to date*
Workers' compensation insurance	*2015*	*\$4,000,000*	*0.76*	*\$1,900,000*
	2016	*\$4,500,000*	*0.75*	*\$1,100,000*
	2017	*\$5,200,000*	*0.77*	*\$700,000*
Fire insurance	*2015*	*\$800,000*	*0.74*	*\$580,000*
	2016	*\$920,000*	*0.74*	*\$675,000*
	2017	*\$880,000*	*0.75*	*\$630,000*
Liability insurance	*2015*	*\$2,000,000*	*0.68*	*\$540,000*
	2016	*\$2,400,000*	*0.67*	*\$520,000*
	2017	*\$2,600,000*	*0.66*	*\$190,000*

   Calculate the loss reserves at the end of 2017.

   We use the expected loss ratios to calculate the expected total payments for each policy year:

Policy Type	Policy Year	Expected Payments	Losses paid to date	Reserves needed
Workers' compensation insurance	2015	\$3,040,000	\$1,900,000	\$1,140,000
	2016	\$3,375,000	\$1,100,000	\$2,275,000
	2017	\$4,004,000	\$700,000	\$3,304,000
Fire insurance	2015	\$592,000	\$580,000	\$12,000
	2016	\$680,800	\$675,000	\$5,800
	2017	\$660,000	\$630,000	\$30,000
Liability insurance	2015	\$1,360,000	\$540,000	\$820,000
	2016	\$1,608,000	\$520,000	\$1,088,000
	2017	\$1,716,000	\$190,000	\$1,526,000

The total reserves are therefore \$10,200,800.

3. *The following table shows the paid losses on claims from one line of business of an insurance company over the past 6 years.*

		Development year					
Accident year	*Earned premiums*	*0*	*1*	*2*	*3*	*4*	*5*
2012	*4,118*	*800*	*790*	*680*	*511*	*151*	*164*
2013	*4,346*	*931*	*799*	*636*	*619*	*197*	
2014	*4,538*	*904*	*921*	*682*	*571*		
2015	*4,417*	*906*	*833*	*706*			
2016	*4,656*	*938*	*930*				
2017	*4,845*	*981*					

Assume that all payments on claims arising from accidents in 2012 have now been settled. Estimate the future payments arising each year from open claims arising from accidents in each calendar year using

(a) The loss development triangle method

We first compute the cumulative loss development:

	Development year					
Accident year	0	1	2	3	4	5
2012	800	1590	2270	2781	2932	3096
2013	931	1730	2366	2985	3182	
2014	904	1825	2507	3078		
2015	906	1739	2445			
2016	938	1868				
2017	981					

If we now take cumulative sums over the columns of this to get total losses before each year, we get the following:

Accident year 2012–	Development year 0	1	2	3	4	5
2012	800	1590	2270	2781	2932	3096
2013	1731	3320	4636	5766	6114	
2014	2635	5145	7143	8844		
2015	3541	6884	9588			
2016	4479	8752				
2017	5460					

The loss development factors are

$$\frac{8752}{4479} = 1.95400759098$$

$$\frac{9588}{6884} = 1.39279488669$$

$$\frac{8844}{7143} = 1.2381352373$$

$$\frac{6114}{5766} = 1.06035379813$$

$$\frac{3096}{2932} = 1.05593451569$$

This gives new cumulative losses

Accident year	Development year 0	1	2	3	4	5
2012						3096
2013					3182	3360
2014				3078	3264	3446
2015			2445	3027	3210	3389
2016		1868	2602	3221	3416	3607
2017	981	1917	2670	3306	3505	3701

The average loss development factors are

$$\frac{1}{5}\left(\frac{1590}{800} + \frac{1730}{931} + \frac{1825}{904} + \frac{1739}{906} + \frac{1868}{938}\right) = 1.95508390893$$

$$\frac{1}{4}\left(\frac{2270}{1590} + \frac{2366}{1730} + \frac{2507}{1825} + \frac{2445}{1739}\right) = 1.39374552311$$

$$\frac{1}{3}\left(\frac{2781}{2270} + \frac{2985}{2366} + \frac{3078}{2507}\right) = 1.23816513007$$

$$\frac{1}{2}\left(\frac{2932}{2781} + \frac{3182}{2985}\right) = 1.06014683269$$

$$\frac{3096}{2932} = 1.05593451569$$

This gives new cumulative losses

	Development year					
Accident year	0	1	2	3	4	5
2012						3096
2013					3182	3360
2014				3078	3263	3446
2015			2445	3027	3209	3389
2016		1868	2604	3224	3417	3609
2017	981	1918	2673	3310	3509	3705

(b) The Bornhuetter-Ferguson method with expected loss ratio 0.81.

Using the loss development factors from part (a), the proportion of total losses in each development year is

Development year	Cumulative Proportion of total losses
0	$\frac{1}{1.954008\times1.392795\times1.238135\times1.060354\times1.055935}=0.265051779045$
1	$\frac{1}{1.392795\times1.238135\times1.060354\times1.055935}=0.517913188257$
2	$\frac{1}{1.238135\times1.060354\times1.055935}=0.721346840353$
3	$\frac{1}{1.060354\times1.055935}=0.893124941355$
4	$\frac{1}{1.055935}=0.947028423772$
5	1

Development year	Cumulative Proportion of total losses	Proportion of total losses
0	0.265051779045	0.265051779045
1	0.517913188257	0.252861409212
2	0.721346840353	0.203433652096
3	0.893124941355	0.171778101002
4	0.947028423772	0.053903482417
5	1	0.052971576228

We get the following table:

Accident	Earned	Expected total	Development year				
year	premiums	payments	1	2	3	4	5
2013	4,346	3520.26					186.47
2014	4,538	3675.78				198.14	194.71
2015	4,417	3577.77			614.58	192.85	189.52
2016	4,656	3771.36		767.22	647.84	203.29	199.77
2017	4,845	3924.45	992.34	798.37	674.13	211.54	207.88

For the average loss development factors, we get

Development year	Cumulative Proportion of total losses	Proportion of total losses
0	0.264770464948	0.264770464948
1	0.51764847558	0.252878010632
2	0.721470245385	0.203821769805
3	0.893299300217	0.171829054832
4	0.947028423772	0.053729123555
5	1	0.052971576228

Accident	Earned	Expected total	Development year				
year	premiums	payments	1	2	3	4	5
2013	4,346	3520.26					186.47
2014	4,538	3675.78				197.50	194.71
2015	4,417	3577.77			614.76	192.23	189.52
2016	4,656	3771.36		768.69	648.03	202.63	199.77
2017	4,845	3924.45	992.41	799.89	674.33	210.86	207.88

4. An actuary is reviewing the following claims data:

No. of closed claims

Acc.	*Development Year*					*Ult.*
Year	*0*	*1*	*2*	*3*	*4*	
2013	*396*	*644*	*804*	*824*	*877*	*1014*
2014	*461*	*806*	*1003*	*1071*		*1163*
2015	*625*	*1022*	*1167*			*1486*
2016	*589*	*1007*				*1592*
2017	*703*					*1758*

Total paid losses on closed claims (000's)

Acc.	*Development Year*				
Year	*0*	*1*	*2*	*3*	*4*
2013	*5,014*	*8,472*	*10,946*	*12,188*	*13,660*
2014	*5,605*	*11,374*	*15,878*	*17,628*	
2015	*8,834*	*13,459*	*20,213*		
2016	*8,938*	*14,971*			
2017	*9,250*				

(a) Calculate tables of percentage of claims closed and cumulative average losses.

Percentage of claims closed

Acc.	Development Year					Ult.
Year	0	1	2	3	4	
2013	39.1	63.5	79.3	81.3	86.5	
2014	39.6	69.3	86.2	92.1		
2015	42.1	68.8	78.5			
2016	37.0	63.3				
2017	40.0					

Cumulative Average Losses:

Acc.	Development Year					Ult.
Year	0	1	2	3	4	
2013	\$12,662	\$13,155	\$13,614	\$14,791	\$15,576	\$17,105
2014	\$12,158	\$14,112	\$15,831	\$16,459	\$15,641	\$18,208
2015	\$14,134	\$13,169	\$17,320	\$16,737	\$16,443	\$15,709
2016	\$15,175	\$14,867	\$15,639	\$16,558	\$16,851	\$18,690
2017	\$13,158	\$14,864	\$18,228	\$16,433	\$16,748	\$18,780

(b) Adjust the total loss table to use the current disposal rate.

The current disposal rates are

Development Year	Disposal rate
0	$\frac{703}{1758} = 0.399886234357$
1	$\frac{1007}{1592} = 0.632537688442$
2	$\frac{1167}{1486} = 0.78532974428$
3	$\frac{1071}{1163} = 0.920894239037$
4	$\frac{877}{1014} = 0.864891518738$

The adjusted losses (in thousands) are therefore

Acc.	Development Year				
Year	0	1	2	3	4
2013	5,134	8,438	10,842	13,812	13,660
2014	5,654	10,381	14,459	17,628	
2015	8,399	12,378	20,213		
2016	9,661	14,971			
2017	9,250				

(c) Use the chain ladder method to estimate claim development based on the adjusted numbers. Compare this to the chain ladder method on aggregate payments on closed claims.

The loss development factors are

Development year	Loss development
0/1	1.6003846
1/2	1.4588797
2/3	1.2426776
3/4	0.9890033

This results in the following estimated cumulative payments

Acc.	Development Year				
Year	0	1	2	3	4
2014					17434
2015				25118	24842
2016			21841	27141	26843
2017		14804	21597	26838	26542

Using the aggregate losses, the loss development factors are

Development year	Loss development
0/1	1.700398
1/2	1.412310
2/3	1.111542
3/4	1.120775

This results in the following estimated cumulative payments

Acc.	Development Year				
Year	0	1	2	3	4
2014					19757
2015				22468	25181
2016			21144	23502	26341
2017		15729	22214	24692	27674

Standard Questions

5. *The number of claims on an insurance policy follows a Poisson distribution with mean 40. For each claim, there is the following distribution of years to settlement and final claim amount:*

Years to settlement	*Probability*	*Final Claim amount*	
		Mean	*Standard Deviation*
0	*0.2*	*800*	*300*
1	*0.3*	*800*	*300*
2	*0.2*	*1,000*	*350*
3	*0.15*	*1,300*	*500*
4	*0.1*	*1,700*	*1,100*
5	*0.05*	*2,800*	*2,300*

(a) Calculate the expected loss development ratio.

The expected loss payments in each year are given by

Year	Expected loss payment
0	$40 \times 0.2 \times 800 = 6400$
1	$40 \times 0.3 \times 800 = 9600$
2	$40 \times 0.2 \times 1000 = 8000$
3	$40 \times 0.15 \times 1300 = 7800$
4	$40 \times 0.1 \times 1700 = 6800$
5	$40 \times 0.05 \times 2800 = 5600$

The expected loss development factors are therefore

Year	Loss Development Factor
0/1	$\frac{16000}{6400} = 2.5$
1/2	$\frac{24000}{16000} = 1.5$
2/3	$\frac{31800}{24000} = 1.325$
3/4	$\frac{38600}{31800} = 1.21383647799$
4/5	$\frac{44200}{38600} = 1.14507772021$

(b) For policies sold 4 years ago, what is the probability that the losses paid out in development year 5 are more than twice the expected lossed using the loss development ratio? You may use a normal approximation for the aggregate losses in a given year.

If the losses paid out in Years 0–4 are X_0, X_1, X_2, X_3 and X_4, and the losses paid out in Year 5 are X_5, then using the loss-development ratio, the expected losses are $0.14507772021(X_0+X_1+X_2+X_3+X_4)$, so we want to find the probability that $X_5 > 0.29015544042(X_0 + X_1 + X_2 + X_3 + X_4)$.

We compute the variance of the aggregate losses in each year:

Year	Variance of loss payment
0	$40 \times 0.2 \times \left(800^2 + 300^2\right) = 5840000$
1	$40 \times 0.3 \times \left(800^2 + 300^2\right) = 8760000$
2	$40 \times 0.2 \times \left(1000^2 + 350^2\right) = 8980000$
3	$40 \times 0.15 \times \left(1300^2 + 500^2\right) = 11640000$
4	$40 \times 0.1 \times \left(1700^2 + 1100^2\right) = 16400000$
5	$40 \times 0.05 \times \left(2800^2 + 2300^2\right) = 26260000$

Using the normal approximation, this means that $X_0 + X_1 + X_2 + X_3 + X_4 \sim N(38600, 51620000)$, while $X_5 \sim N(5600, 26260000)$. Therefore $X_5 - 0.29015544042(X_0 + X_1 + X_2 + X_3 + X_4) \sim N(5600 - 0.29015544042 \times 38600, 26260000 + 0.29015544042 \times 51620000) = N(-5600, 41237823.8345)$. The probability that $X_5 > 0.29015544042(X_0 + X_1 + X_2 + X_3 + X_4)$ is therefore $1 - \Phi\left(\frac{5600}{\sqrt{41237823.8345}}\right) = 0.1915912$.