ACSC/STAT 4703, Actuarial Models II
Fall 2018
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Homework Sheet 7
Model Solutions

Basic Questions

1. *An insurance company has the following data on its policies:*

Policy limit	*Losses Limited to*			
	20,000	*50,000*	*100,000*	*500,000*
20,000	*2,500,000*			
50,000	*9,360,000*	*10,200,000*		
100,000	*21,800,000*	*24,200,000*	*28,060,000*	
500,000	*4,390,000*	*5,020,000*	*5,880,000*	*6,060,000*

Use this data to calculate the ILF from \$20,000 to \$500,000 using

(a) The direct ILF estimate.

For policies with policy limit at least \$500,000, claims on losses limited to \$20,000 are \$4,390,000, while claims from losses limited to \$500,000 are \$6,060,000, so the direct ILF estimate is $\frac{6060000}{4390000} = 1.38041002278$

(b) The incremental method.

The ILF from \$20,000 to \$50,000 is $\frac{39420000}{35550000} = 1.10886075949$. The ILF from \$50,000 to \$100,000 is $\frac{33940000}{29220000} = 1.16153319644$. The ILF from \$100,000 to \$500,000 is $\frac{6060000}{5880000} = 1.0306122449$. Therefore, the ILF from \$20,000 to \$500,000 is $1.10886075949 \times 1.16153319644 \times 1.0306122449 = 1.32740649817$.

2. *For a certain line of insurance, the loss amount per claim follows a Pareto distribution with parameters $\alpha = 3$ and θ. If the policy has a deductible per loss set at 0.1θ and a policy limit set at 4θ, by how much will the expected payment per loss increase if there is inflation of 3%?*

The expected payment per loss is

$$\begin{aligned}
\int_{0.1\theta}^{4\theta} \left(\frac{\theta}{\theta + x}\right)^3 dx &= \theta \int_{1.1}^{5} u^{-3}\, du \\
&= \theta \left[-\frac{u^{-2}}{2}\right]_{1.1}^{5} \\
&= \frac{1}{2}\left(\frac{1}{1.1^2} - \frac{1}{5^2}\right)\theta \\
&= 0.3932231404496\theta
\end{aligned}$$

where we have made the substitution $u = \frac{x+\theta}{\theta}$

After inflation the loss is Pareto with parameters $\alpha = 3$ and 1.03θ. The expected payment per loss is therefore

$$\begin{aligned}
\int_{0.1\theta}^{4\theta} \left(\frac{1.03\theta}{1.03\theta + x}\right)^3 dx &= 1.03\theta \int_{\frac{1.13}{1.03}}^{\frac{5.03}{1.03}} u^{-3}\, du \\
&= 1.03\theta \left[-\frac{u^{-2}}{2}\right]_{\frac{1.13}{1.03}}^{\frac{5.03}{1.03}} \\
&= \frac{1.03}{2}\left(\left(\frac{1.03}{1.13}\right)^2 - \left(\frac{1.03}{5.03}\right)^2\right)\theta \\
&= 0.4062881358873\theta
\end{aligned}$$

where we have made the substitution $u = \frac{x+1.03\theta}{1.03\theta}$.

The expected payment will therefore increase by $\frac{0.4062881358873\theta - 0.3932231404496\theta}{0.3932231404496\theta} =$ 3.3225%.

3. *An insurance company charges a risk charge equal to the square of the average loss amount, divided by 20,000. It has the following data on a set of 600 claims from policies with limit $1,000,000.*

Losses Limited to	*20,000*	*50,000*	*100,000*	*500,000,1,000,000*	
Total claimed	*9,350,000*	*11,630,000*	*13,380,000*	*14,400,000*	*15,020,000*

Calculate the ILF from $100,000 to $1,000,000.

Standard Questions

4. *An insurer calculates the ILF from $500,000 to $1,000,000 on a particular policy is 1.081. The average loss per unit of exposure with the policy limit of $1,000,000 is $1,329. The insurer's premium also includes a risk charge equal to the square of the expected loss divided by 5,000. A reinsurer is willing to provide excess-of-loss reinsurance of $500,000 over $500,000 (that is, the attachment point is $500,000 and the limit on the reinsurer's payment is $500,000) with a loading of 20%.*

   (a) Calculate the average loss per unit of exposure for a policy with limit $500,000

   The premium for insurance with policy limit $1,000,000 is $1329 + \frac{1329^2}{5000} =$ $\$1,682.2482$. Therefore the premium with policy limit $500,000 is $\frac{1682.2482}{1.081} =$ $\$1,556.19629972$. The expected payment with policy limit $500,000 is therefore the solution to

$$
\begin{aligned}
x + \frac{x^2}{5000} &= 1556.19629972 \\
x^2 + 5000x - 7780981.4986 &= 0 \\
x &= \frac{-5000 + \sqrt{5000^2 + 4 \times 7780981.4986}}{2} \\
x &= \$1,245.79517574
\end{aligned}
$$

(The other solution to the quadratic is negative, and so is not a possible answer.)

Thus, the average loss per unit of exposure is \$1,245.79517574.

(b) Calculate the premium the insurance company should charge for a policy with limit \$1,000,000 if they buy excess-of-loss reinsurance.

The expected payment by the reinsurer is $1329 - 1245.79517574 = 83.20482426$. Therefore the reinsurer's premium is $1.2 \times 83.20482426 = 99.845789112$. Therefore the insurer should charge $1556.19629972 + 99.845789112 = \$1,656.04$ for the policy if they buy reinsurance.

5. *An insurer computes a trend factors of 1.059 for policies with limit \$500,000. The insurance company buys excess-of-loss reinsurance of \$500,000 over \$500,000 on its policies with policy limit \$1,000,000. The loading on this reinsurance is 30%. The reinsurance premium is 5% of the insurer's expected loss payments. After the trend factors are applied, the reinsurer's loading decreases to 25%, and the reinsurance premium becomes 5.3% of insurer's expected losses. What is the trend factor for policy limit \$1,000,000?*

   Let the insurer's expected loss payments be x. The reinsurance premium is $0.05x$. Since the loading is 30%, the expected reinsurance payments are $\frac{0.05x}{1.3} = 0.0384615384615x$. Therefore the expected total payments are $1.0384615384615x$. After applyin the trend factor, the insurer's expected loss payment is $1.059x$, so the reinsurance premium is $1.059 \times 0.053x = 0.056127x$. Since the reinsurer's loading has reduced to 25%, the expected reinsurer's payment is $\frac{0.056127x}{1.25} = 0.0449016x$. The total payment is therefore $1.059x + 0.0449016x = 1.1039016x$, so the trend factor for policy limit \$1,000,000 is $\frac{1.1039016x}{1.0384615384615x} = 1.06301635556$.