ACSC/STAT 4703, Actuarial Models II
WINTER 2020
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Midterm Examination
Monday 2nd March, 13:35-14:25 AM

Each part question (a, b, c, etc.) is worth 1 mark. You should have been provided with a formula sheet. **No other notes are permitted**. Scientific calculators are permitted, but not graphical calculators.

Here are some values of the Gamma distribution function with $\theta = 1$ that may be needed for this examination:

x	α	$F(x)$	x	α	$F(x)$	x	α	$F(x)$
245	255	0.2697208	2.5	4	0.2424239	4.375	4	0.6361773
$\left(\frac{7.5}{12}\right)^3$	$\frac{4}{3}$	0.1117140	3.875	3	0.7430029	4.875	4	0.7169870
$\left(\frac{9.5}{12}\right)^3$	$\frac{4}{3}$	0.2507382	4.375	3	0.8118663	5.375	4	0.7837292
2.5	1	0.917915	4.875	3	0.8644174	2.156	5	0.06782354
2.5	2	0.7127025	5.375	3	0.9035828	3.203	5	0.219922
2.5	3	0.4561869	3.875	4	0.5417358	8.542	5	0.9274742

1. Using an arithmetic distribution ($h = 1$) to approximate a Weibull distribution with $\tau = 2$ and $\theta = 7$, calculate the probability that the value is more than 6.5, for the approximation using the method of local moment matching, matching 1 moment on each interval.

   [Hint:

$$\int_{5.5}^{7.5} \frac{2}{49}x^2 e^{-\left(\frac{x}{7}\right)^2}\,dx = 1.435464$$
$$\int_{6}^{7} \frac{2}{49}x^2 e^{-\left(\frac{x}{7}\right)^2}\,dx = 0.7254893$$

   ]

2. Claim frequency follows a Poisson distribution with $\lambda = 2.4$. Claim severity (in thousands) has the following distribution:

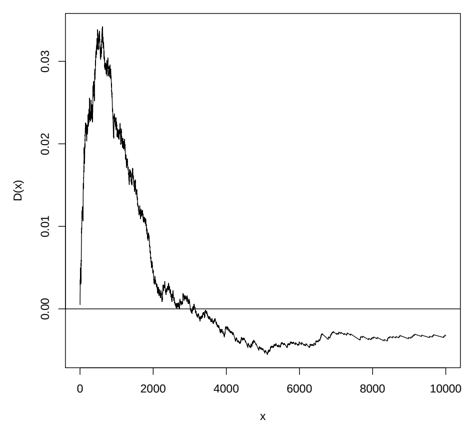
Severity	Probability
0	0.21
1	0.44
2	0.32
3	0.03

   The company buys excess-of loss reinsurance for aggregate losses exceeding 2.

(a) Use the recursive method to calculate the probability that the reinsurance makes a payment.

(b) What is the expected payment on the reinsurance? [Hint: Consider the difference between the expected aggregate losses and the expected payments made with reinsurance.]

3. An insurance company collects a sample of 6000 claims. Based on previous experience, it believes these claims might follow a Pareto distribution with $\alpha = 3$ and $\theta = 2000$. To test this, it computes the following plot of $D(x) = F_n(x) - F^*(x)$.



(a) How many of the claims in their sample were more than 3,000?

(b) Which of the following statements best describes the fit of the Pareto distribution to the data:

(i) The Pareto distribution assigns too much probability to high values and too little probability to low values.

(ii) The Pareto distribution assigns too much probability to low values and too little probability to high values.

(iii) The Pareto distribution assigns too much probability to tail values and too little probability to central values.

(iv) The Pareto distribution assigns too much probability to central values and too little probability to tail values.

Justify your answer.

4. An insurance company collects the following sample:

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21.23 23.88 83.10 86.25 226.15 381.31 458.78 606.75
1201.73 1857.35
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They model this as following a distribution with the following distribution function:

x	$F(x)$	$\log(F(x_{i+1})) - \log(F(x_i))$	$\log(1 - F(x_i)) - \log(1 - F(x_{i+1}))$
21.23	0.07957669	0.20005185	0.01933289
23.88	0.09720023	1.35724389	0.37202795
83.10	0.37766854	0.02503241	0.01550247
86.25	0.38724181	0.46555216	0.46950458
226.15	0.61683496	0.14798137	0.29673049
381.31	0.71521477	0.04070322	0.11018515
458.78	0.74492690	0.05233498	0.17068346
606.75	0.78495083	0.09206273	0.43385253
1201.73	0.86064646	0.03942506	0.28548762
1857.35	0.89525524	0.11064642	NA

Calculate the Kolmogorov-Smirnov statistic for this model and this data.

5. An insurance company collects a sample of 700 claims. They want to decide whether this data is better modeled as following an inverse exponential, or a transformed beta distribution. After calculating MLE estimates for the parameters (1 parameter for the inverse exponential and 4 for the transformed beta), log-likelihoods for the two distributions are:

Distribution	log-likelihood
Inverse Exponential	-4341.82
Transformed Beta	-4334.55

Use the Bayes Information Criterion (BIC) to decide which distribution is a better fit for the data.

6. A homeowner's house is valued at \$340,000. However, the home is insured only to a value of \$190,000. The insurer requires 70% coverage for full insurance. The home sustains \$8,000 of damage from a break-in. The deductible is \$4,000, decreasing linearly to zero for losses of \$10,000. How much does the insurer reimburse?

7. The following table shows the cumulative losses (in thousands) on claims from one line of business of an insurance company over the past 4 years.

	Development year			
Accident year	0	1	2	3
2016	645	1021	1098	1307
2017	729	1100	1123	
2018	804	1210		
2019	751			

Using the mean for calculating loss development factors, esimate the total reserve needed for payments to be made in 2020 using the Bornhuetter-Fergusson method. The expected loss ratio is 0.72 and the earned premiums in each year are given in the following table:

Year	Earned Premiums (000's)
2016	1857
2017	1944
2018	2143
2019	2095

[Assume no more payments are made after development year 3.]