ACSC/STAT 4703, Actuarial Models II
Fall 2020

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Homework Sheet 2

Due: Monday 10th February: 13:30 PM

Basic Questions

1. An insurance company has the following portfolio of auto insurance policies:

Type of driver	Number	Probability of claim	mean claim	standard deviation
Learner	1100	0.035	\$3,900	\$34,800
Normal	700	0.015	\$3,400	\$31,400
Advanced	400	0.011	\$2,800	\$32,100

   Calculate the cost of reinsuring losses above \$1,000,000, if the loading on the reinsurance premium is one standard deviation above the expected claim payment on the reinsurance policy using a Pareto approximation for the aggregate losses on this portfolio.

2. An insurance company is modelling claim data as following a Pareto distribution with $\alpha = 2.6$. It collects the following sample of claims:

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0.1 0.3 0.7 1.5 4.7 6.7 6.9 7.4 7.8 14.0 20.6 20.8
22.1 24.3 38.1 44.7 70.7 157.0 244.1 254.6 280.1 282.0
285.3 424.8 928.3 1119.9 1694.3 2792.2 2979.6 3613.8
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   The MLE for θ is 405.5201. Graphically compare this empirical distribution with the best fitting Pareto distribution with $\alpha = 2.6$. Include the following plots:

   (a) Comparisons of $F(x)$ and $F^*(x)$

   (b) Comparisons of $f(x)$ and $f^*(x)$

   (c) A plot of $D(x)$ against x.

   (d) A p-p plot of $F(x)$ against $F^*(x)$.

3. For the data in Question 2, calculate the following test statistics for the goodness of fit of the Pareto distribution with $\alpha = 2.6$ and $\theta = 405.5201$:

   (a) The Kolmogorov-Smirnov test.

   (b) The Anderson-Darling test.

   (c) The chi-square test, dividing into the intervals 0–100, 100–500, and more than 500.

4. For the data in Question 2, perform a likelihood ratio test to determine whether a Pareto distribution with fixed $\alpha = 2.6$, or a Pareto distribution with α freely estimated is a better fit for the data. [The MLE for the general Pareto distribution is $\alpha = 0.4254277$ and $\theta = 9.096$.]

5. For the data in Question 2, use AIC and BIC to choose between a Pareto distribution with $\alpha = 2.6$ for the data and a gamma distribution. [The MLE for the gamma distribution is $\alpha = 0.2962591$ and $\theta = 1726.7992261$.]

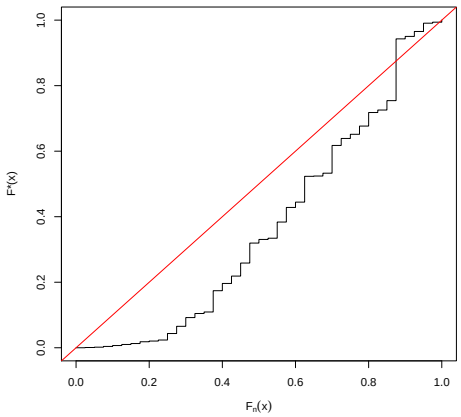
Standard Questions

6. An inland marine insurance company insures three types of vehicles and has the following estimates:

Property type	Probability of claim	mean claim	standard deviation
Train	0.01	\$35,600	\$594,800
Ship	0.03	\$21,300	\$334,900
Truck	0.13	\$8,600	\$217,300

The insurance company estimates the mean μ and standard deviation σ for the aggregate loss distribution, and buys stop-loss insurance for losses above \$2,500,000. One reinsurer models aggregate losses as following a Pareto distribution and sets its premium as 120% of the expected claims on the stop-loss policy. Another reinsurer models aggregate losses as following a Gamma distribution, and sets its premium at 150% of the expected claims. The company insures 134 trains, 211 ships and 403 trucks. Which reinsurance company is cheaper?

7. An insurance company collects a sample of 40 past claims, and attempts to fit a distribution to the claims. Based on experience with other claims, the company believes that a Gamma distribution with $\alpha = 2$ and $\theta = 1,400$ may be appropriate to model these claims. It constructs the following p-p plot to compare the sample to this distribution:



(a) How many of the points in their sample were less than 2,300?

(b) Which of the following statements best describes the fit of the Gamma distribution to the data:

(i) The Gamma distribution assigns too much probability to high values and too little probability to low values.

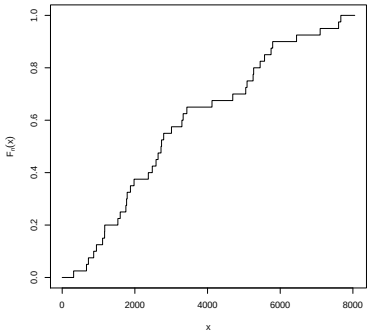
(ii) The Gamma distribution assigns too much probability to low values and too little probability to high values.

(iii) The Gamma distribution assigns too much probability to tail values and too little probability to central values.

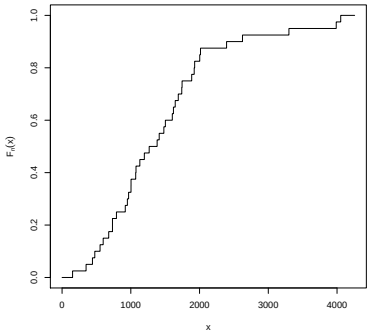
(iv) The Gamma distribution assigns too much probability to central values and too little probability to tail values.

(c) Which of the following plots shows the empirical distribution function? Justify your answer.

(i)



(ii)



(iii)