ACSC/STAT 4703, Actuarial Models II
Fall 2020

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Homework Sheet 6

Model Solutions

Basic Questions

1. *An insurance company starts a new line of insurance in 2019, and collects a total of \$2,600,000 in premiums that year, and the estimated incurred losses for accident year 2019 are \$982,000. The premium payments are uniformly distributed over the year. An actuary is using this data to estimate rates for premium year 2022. Claims are subject to 5% inflation per year. By what percentage should premiums increase from 2019 in order to achieve a loss ratio of 0.75.*

With premium payments uniformly distributed throughout the year, the average portion of the year covered by each premium is $\frac{1}{2}$, so the earned premiums for the year is \$1,300,000. The incurred losses of \$982,000 give an achieved loss ratio of $\frac{982000}{1300000} = 0.755384615385$. This means that without inflation the premium would be adjusted by a factor of $\frac{0.755384615385}{0.75} = 1.00717948718$. The average inflation from the start of accident year 2019 to the claim time is $\int_0^1 2t(1.05)^t\,dt$. We have

$$\begin{aligned}\int_0^1 t(1.05)^t\,dt &= \left[t\frac{(1.05)^t}{\log(1.05)}\right]_0^1 - \int_0^1 \frac{(1.05)^t}{\log(1.05)}\,dt\\ &= \frac{(1.05)}{\log(1.05)} - \frac{0.05}{\log(1.05)^2}\\ &= 0.5165648592\end{aligned}$$

so $\int_0^1 2t(1.05)^t\,dt = 2\times 0.5165648592 = 1.0331297184$. The average inflation from the start of policy year 2022 to claim time is

$$\int_0^1 t(1.05)^t\,dt+\int_1^2 (2-t)(1.05)^t\,dt = \int_0^1 (t+1.05(1-t))(1.05)^t = 1.05\int_0^1 (1.05)^t\,dt - 0.05\int_0^1 t(1.05)^t\,dt$$

Thus

$$1.05\int_0^1 (1.05)^t\,dt - 0.05\int_0^1 t(1.05)^t\,dt = \frac{0.05^2}{\log(1.05)^2} = 1.05020830854$$

Therefore the inflation that needs to be applied to the premium is $\frac{1.05^3\times 1.05020830854}{1.0331297184} = 1.17676161233$. Therefore the premium needs to be adjusted by a factor $1.00717948718\times 1.17676161233 = 1.18521015724$.

2. *An insurer collects \$900,000 in earned premiums for accident year 2019. The total loss payments are \$552,000. Payments are subject to inflation of 6%, and policies are sold uniformly throughout the year. If the insurer's permissible loss ratio is 75%, by how much should the premium be changed for policy year 2021?*

   The loss ratio from 2019 is $\frac{552000}{900000} = 0.613333333333$, so without inflation the loss ratio should be adjusted by a factor $\frac{0.613333333333}{0.75} = 0.817777777777$. The average inflation from the start of accident year 2019 to the claim time is $\int_0^1 (1.06)^t\,dt = \frac{0.06}{\log(1.06)} = 1.02970867194$. As in (a), we get

   $$\int_0^1 t(1.06)^t\,dt + \int_1^2 (2-t)(1.06)^t\,dt = \frac{0.06^2}{\log(1.06)^2} = 1.06029994908$$

   Therefore the policy needs to be adjusted by inflation of

   $$\frac{1.06029994908(1.06)^2}{1.02970867194} = 1.15698066381$$

   The total adjustment factor is therefore $1.15698066381 \times 0.817777777777 = 0.946153076182$.

3. *An auto insurer has three lines of insurance — motorcycle, car and truck. The experience from policy year 2019 is:*

Sector	*Current differential*	*Earned premiums*	*Loss payments*
Motocycle	*1.2*	*2,100*	*1,500*
Car	*1*	*8,230*	*6,400*
Truck	*2.3*	*4,050*	*3,400*

   The base premium was \$840. Claim amounts are subject to 4% annual inflation. If the expense ratio is 30%, calculate the new premiums for each sector for policy year 2021.

   The loss ratios are

Sector	Current differential	Loss ratio	New differential
Motocycle	1.2	$\frac{1500}{2100} = 0.714285714286$	$1.2 \times \frac{0.714285714286}{0.777642770352} = 1.10223214286$
Car	1	$\frac{6400}{8230} = 0.777642770352$	1
Truck	2.3	$\frac{3400}{4050} = 0.83950617284$	$2.3 \times \frac{0.83950617284}{0.777642770352} = 2.48297067902$

   Using these differentials, the adjusted earned premiums are $2100 \times \frac{0.714285714286}{0.777642770352} = 1928.90625$ and $4050 \times \frac{0.83950617284}{0.777642770352} = 4372.18750002$, so the total adjusted earned premiums are $1928.90625 + 8230 + 4372.18750002 = 14531.09375$. The loss ratio is therefore $\frac{11300}{14531.09375} = 0.777642770352$, so without inflation, the base premium should change by a factor of $\frac{0.777642770352}{0.7} = 1.11091824336$. The inflation from policy year 2019 to policy year 2021 is $(1.04)^2$, so the new base premium needs to be $840 \times 1.11091824336(1.04)^2 = \$1,009.32$. With the new differentials, the premiums are:

Car	$\$1,009.32$
Motorcycles	$1009.3181045 \times 1.10223214286 = \$1,112.50$
Trucks	$1009.3181045 \times 2.48297067902 = \$2,506.11$

Standard Questions

4. *An insurer has different premiums for residential and commercial properties. Its experience for accident year 2019 is given below. There was a rate change on 6th June 2019 [157th day of the year], which affects some of the policies.*

Policy Type	*Differential before rate change*	*Current differential*	*Earned premiums*	*Loss payments*
Residential	*1*	*1*	*15,400*	*12,200*
Commercial	*1.31*	*1.22*	*12,700*	*8,400*

Before the rate change, the base premium was $1140. The current base premium is $1310. Assuming that policies are sold uniformly over the year, calculate the new premimums for policy year 2021 assuming 4% annual inflation and a permissible loss ratio of 0.8.

The new policy came into effect $\frac{157}{365}$ of the year, proportion of earned premiums covered by the new premiums is $\frac{1}{2}\left(1-\frac{157}{365}\right)^2 = 0.162371927191$. To adjust the premiums to the old premium, we multiply the the earned premiums by $\frac{1140}{1140\times 0.837628072809+1310\times 0.162371927191} = 0.97635906876$ for the base class, and by $\frac{1140\times 1.31}{1140\times 1.31\times 0.837628072809+1310\times 1.22\times 0.162371927191} = 0.988733851165$ for commercial properties. The adjusted earned premiums are therefore $15400 \times 0.97635906876 = 15035.9296589$ for residential properties and $12700 \times 0.988733851165 = 12556.9199098$ for commercial properties.

(If we adjust to the new premiums we get $15400\times\frac{1310}{1140\times 0.837628072809+1310\times 0.162371927191} = 17278.1296958$ for residential properties and $12700\times\frac{1310\times 1.22}{1140\times 1.31\times 0.837628072809+1310\times 1.22\times 0.162371927191} = 13438.1072719$ for commercial properties.)

With these adjusted earned premiums, the loss ratios are $\frac{12200}{15035.9296589} = 0.811389802743$ for residential properties and $\frac{8400}{12556.9199098} = 0.668953856546$ for commercial properties. This means the new differential is $1.31 \times \frac{0.668953856546}{0.811389802743} = 1.08003520517$. With this differential, the adjusted earned premiums would be $\frac{0.668953856546}{0.811389802743} \times 12556.9199098 = 10352.6073061$. This makes the total adjusted earned premiums $15035.9296589+10352.6073061 = 25388.536965$, so the adjusted loss ratio is $\frac{20600}{25388.536965} = 0.811389802744$. The premium therefore needs to be adjusted by a factor of $\frac{0.811389802744}{0.8} = 1.01423725343$, to $1140 \times 1.01423725343 = 1156.23046891$. From accident year 2019 to policy year 2021, inflation is $(1.04)^2\frac{\left(\frac{0.04^2}{\log(1.04)^2}\right)}{\left(\frac{0.04}{\log(1.04)}\right)} = (1.04)^2\frac{0.04}{\log(1.04)} = 1.10309059988$

The new base premium is $1156.23046891\times 1.10309059988 = 1275.42696155$, and the new premium for commercial properties is $1275.42696155\times 1.08003520517 = \$1,377.51$.

5. *An insurer classifies inland marine insurance policyholders into truck or*

train, and into low-risk or high-risk. It has the following data from policy year 2019:

	Number of policies	
	low-risk	*high-risk*
Train	*230*	*32*
Truck	*252*	*844*

	loss payments	
	low-risk	*high-risk*
Train	*\$150,400*	*\$44,200*
Truck	*\$311,300*	*\$2,042,000*

(a) If the base classes are Truck and high-risk, the base rate is \$3,240, and the differentials are 0.5 for Train and 0.4 for low-risk, calculate the new premiums which give an expense ratio of 0.3 using the loss-ratio method.

Multiplying the number of policies by the premium gives the annual earned premiums

	low-risk	high-risk
Train	$230 \times 3240 \times 0.5 \times 0.4 = 149,040$	$32 \times 3240 \times 0.5 = 51,840$
Truck	$252 \times 3240 \times 0.4 = 326,592$	$844 \times 3240 = 2,734,560$

The loss ratios for Train and Truck are therefore $\frac{194600}{200880} = 0.968737554759$ and $\frac{2353300}{3061152} = 0.768762870971$ respectively, so the new differential for Train is $0.5 \times \frac{0.968737554759}{0.768762870971} = 0.63006265738$. The loss ratios for low-risk and high-risk are $\frac{461700}{475632} = 0.970708446866$ and $\frac{2086200}{2786400} = 0.748708010336$ respectively. The new differential for low risk is therefore $\frac{0.970708446866}{0.748708010336} \times 0.4 = 0.51860454728$. Using these differentials to balance back, with these differentials at the current base premium, we get total earned premiums of

$$3240\left(844 + 32 \times 0.63006265738 + 252 \times 0.51860454728 + 230 \times 0.51860454728 \times 0.63006265738\right)$$
$$=3466811.74033$$

and the loss ratio would be $\frac{2547900}{3466811.74033} = 0.734940397934$. The base premium for 2019 therefore needs to be adjusted by a factor $\frac{0.734940397934}{0.70} = 1.04991485419$. So the new base premium is $3240 \times 1.04991485419 = 3401.72412758$. The new premiums are:

low-risk trucks	$3401.72412758 \times 0.51860454728 = \$1,764.15$
high-risk trains	$3401.72412758 \times 0.63006265738 = \$2,143.30$
low-risk trains	$3401.72412758 \times 0.63006265738 \times 0.51860454728 = \$1,111.52$

(b) Repeat part (a) based on differentials of 1.6 for Train and 0.8 for low-risk.

Multiplying the number of policies by the premium gives the annual earned premiums

	low-risk	high-risk
Train	$230 \times 3240 \times 1.6 \times 0.8 = 953,856$	$32 \times 3240 \times 1.6 = 165,888$
Truck	$252 \times 3240 \times 0.8 = 653,184$	$844 \times 3240 = 2,734,560$

The loss ratios for Train and Truck are therefore $\frac{194600}{1119744} = 0.173789723365$ and $\frac{2353300}{3387744} = 0.694651071628$ respectively, so the new differential for

Train is $1.6 \times \frac{0.173789723365}{0.694651071628} = 0.400292418368$. The loss ratios for low-risk and high-risk are $\frac{461700}{1607040} = 0.287298387097$ and $\frac{2086200}{2900448} = 0.71926819581$ respectively. The new differential for low risk is therefore $\frac{0.287298387097}{0.71926819581} \times 0.8 = 0.319545214173$. Using these differentials to balance back, with these differentials at the current base premium, we get total earned premiums of

$$3240\,(844 + 32 \times 0.400292418368 + 252 \times 0.319545214173 + 230 \times 0.319545214173 \times 0.400292418368)$$
$$=3132284.264$$

and the loss ratio would be $\frac{2547900}{3132284.264} = 0.81343191909$. The base premium for 2019 therefore needs to be adjusted by a factor $\frac{0.81343191909}{0.70} = 1.1620455987$. So the new base premium is $3240 \times 1.1620455987 = 3765.02773979$.

	low-risk trucks	$3765.02773979 \times 0.319545214173 = \$1,203.10$
The premiums are:	high-risk trains	$3765.02773979 \times 0.400292418368 = \$1,507.11$
	low-risk trains	$3765.02773979 \times 0.319545214173 \times 0.400292418368 = \$481.59$