Banff International Research Station

for Mathematical Innovation and Discovery

Emergent behaviour in multi-particle systems with non-local interactions

22-27 January, 2012

MEALS

All meals are served in Sally Borden Building. Breakfast is 7:00-9:30; lunch is 11:30-1:30; dinner: is 5:30-7:30. On Sunday, only dinner is included; on Friday, only breakfast and lunch. **Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector, overhead projectors and blackboards are available for presentations.

SCHEDULE

Notes: The total time allocated to speakers is indicated in parentheses. Please allow 5 minutes for questions/changeover.

Monday:

8:45-9:00	Introduction
9:00-10:40	Levy(40), Lega(30), Bernoff(30)
10:30-11:00	Coffee
11:00-12:00	Gazi(30), Haskovec(30)
12:00-1:00	Lunch
1:00-2:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
2:00	Group photo, meet on the front steps of Corbett Hall
2:10-3:20	Eftimie(40), Birnir(30)
3:20-3:40	Coffee
3:40-4:50	Einarsson(20), Barbaro(20), Ward(30)
4:50-5:30	Discussion
5:30-7:30	Dinner
7:30-8:30	Lindsay(20), Rodriguez(20), Rosado(20)

Tuesday:

9:00-10:20	Martin(30), Fetecau(30), Huang(20)
10:20-10:45	Coffee
10:45-12:00	Bertozzi(45), Laurent(30)
12:00-2:00	Lunch
2:00-3:10	Panferov(30), Pavlovsky(20), Kolokolnikov(20)
3:10-3:40	Coffee
3:40-4:30	Discussion
4:30-5:20	Yao(20), Raoul(20), Bedrossian(20)
5:20-7:30	Dinner
7:30-8:30	Discussion

Wednesday

9:00-10:20	Carrillo(40), Balagué (20), Motsch(20)
10:20-10:40	Coffee
10:40-12:00	Frouvelle(20), Agueh(30), Ha(30)
12:00	Lunch
	Afternoon/evening free

Thursday:

9:00-10:40	Erban(40), deVries(30), Wennberg(30)
10:40-11:00	Coffee
11:00-12:00	Putkaradze(30), D'Orsogna(30)
12:00-1:40	Lunch
1:40-3:10	Fellner(30), Escudero(30), Forgoston (30)
3:10-3:40	Coffee
3:40-5:30	Open problems/discussion
5:30-7:30	Dinner
7:30-8:30	Discussion

Friday:

9:00-13:00	Discussion
10:30-11:00	Coffee; **Checkout by noon.**
12:00	Lunch.

ABSTRACTS

Martial Agueh

A refined flocking and swarming model of Cucker-Smale type

The Cucker-Smale model is a mathematical model of swarming that leads to (unrealistic!) unconditional flocking of all the birds in the swarm. In this work, we generalize this model to scenarios where a typical bird is subject to a friction force driving it to fly at optimal speed, a repulsive short-range force to avoid collisions, an attractive "flocking" force which takes into account a cone of vision of the bird, and a boundary force to bring the bird back inside the swarm if it is on the edge flying outward. We present the particle model system, derive the associated kinetic equation, and study its well-posedness. Finally, we show some numerical simulations of the model. Our simulations confirm that breakup of a swarm does occur, contrarily to the standard Cucker-Smale model.

This is a joint work with R. Illner and A. Richardson.

Daniel Balagué

Stationary states for the aggregation equation with power law attractive-repulsive potentials

We analyze the stability of a uniform distribution on a sphere as stationary solution of the aggregation equation with power law attractive-repulsive potentials. We give a sharp condition that establishes its stability under radial perturbations.

Alethia Barbaro

A phase transition in a kinetic Cucker-Smale model with friction

We consider the steady-states of a kinetic Cucker-Smale model with friction. As we vary the noise and friction coefficients, we find that a phase transition occurs. For small values of the noise coefficient, there are three steady-state solutions while for larger values, there is only one.

Jacob Bedrossian

Global Existence and Finite Time Blow-Up for Critical Patlak-Keller-Segel Models with Inhomogeneous Diffusion

The L^1-critical parabolic-elliptic Patlak-Keller-Segel system is a classical model of chemotactic aggregation in micro-organisms well-known to have critical mass phenomena. In this paper we study this critical mass phenomenon in the context of Patlak-Keller-Segel models with spatially varying diffusivity of the chemo-attractant in three dimensions and higher. The critical mass is identified to depend only on the local value of the diffusivity and finite time blow-up results show it to be sharp under certain conditions. The methods also provide new blow-up results for homogeneous problems, showing that there exist blow-up solutions with arbitrarily large (positive) initial free energy.

Andrew J. Bernoff

A Primer of Swarm Equilibria.

We study equilibrium configurations of swarming biological organisms subject to exogenous and pairwise endogenous forces. Beginning with a discrete dynamical model, we derive a variational description of the continuum population density. Equilibrium solutions are extrema of an energy functional, and satisfy a Fredholm integral equation. We find conditions for the extrema to be local minimizers, global minimizers, and minimizers with respect to infinitesimal Lagrangian displacements of mass. In one spatial dimension, for a variety of exogenous forces, endogenous forces, and domain configurations, we find exact analytical expressions for the equilibria. These agree closely with numerical simulations of the underlying discrete model. The exact solutions provide a sampling of the wide variety of equilibrium configurations possible within our general swarm modeling framework. The equilibria typically are compactly supported and may contain d-concentrations or jump discontinuities at the edge of the support. In two-dimensions we show that the Morse Potential and other "pointy" potentials can generically lead to inverse square-root singularities in the density at the boundary of the swarm support.

Joint work with Louis Ryan and Chad M. Topaz.

Andrea L. Bertozzi

Swarming and Aggregation Equations

Abstract: This lecture is an introduction to the interesting phenomena of swarming and to the open problems in this area. I will review numerical and analytical results for both kinematic and dynamic aggregation equations. I will discuss how models are constructed and the emergence of phenomenological behavior for different types of models including flocking, milling, and other patterns. I will also review some results on well-posedness of aggregation equations including a sharp condition on blowup from smooth initial data. This talk will include joint work with Bedrossian, Brandman, Carrillo, Laurent, Rodriguez, and Slepcev.

Bjorn Birnir

Dynamic Energy Budget Theory and the Environment

Dynamics Energy Budget Theory (DEB) can be used to model the physiology of animals and how it influences their interactions with the environment. We explain with the example of the Icelandic capelin how changes in physiology can trigger entirely different group behavior influencing migration patterns over large distances.

Jose Antonio Carrillo

A review of 2nd order models for swarming

I will present several examples of the derivation by means of kinetic theory arguments of kinetic equations for swarming. All of them arise from individual based models in the recent literature. For instance, the system of interacting, self-propelled discrete particles by D'Orsogna etal. Starting from the particle model, one can construct solutions to a Vlasov-like kinetic equation for the single particle probability distribution function using distances between measures. Another example is the continuous kinetic version of flocking by Cucker and Smale.

The large-time behavior of the distribution in phase space is subsequently studied by means of particle approximations and a stability property in distances between measures. A continuous analogue of the theorems of Cucker-Smale will be shown to hold for the solutions on the kinetic model. More precisely, the solutions concentrate exponentially fast their velocity to their mean while in space they will converge towards a translational flocking solution. The mean field limit with/without noise for these models will also be discussed. Issues related to particular solutions such as flocks and mills will also be investigated. Hydrodynamic systems will also be formally derived.

Gerda de Vries

Formation of Animal Groups: The Importance of Communication

We investigate the formation and movement of self-organizing collectives of individuals in homogeneous environments. We review a hyperbolic system of conservation laws based on the assumption that the interactions governing movement depend not only on distance between individuals, but also on whether neighbours move towards or away from the reference individual. The inclusion of direction-dependent communication mechanisms significantly enriches the model behavior; the model exhibits classical patterns such as stationary pulses and traveling trains, but also novel patterns such as zigzag pulses, breathers, and feathers. The same enrichment of model behavior is observed when we include direction-dependent communication mechanisms in individual-based models.

Maria D'Orsogna

Stochastic nucleation and growth of particle clusters

The binding of individual components to form composite structures is a ubiquitous phenomenon within the sciences. Within heterogeneous nucleation, particles may be attracted to an initial exogenous site: the formation of droplets, aerosols and crystals usually begins around impurities or boundaries. Homogeneous nucleation on the other hand describes identical particles spontaneously clustering upon contact.

Given their ubiquity in physics, chemistry and material sciences, nucleation and growth have been extensively studied in the past decades, often assuming infinitely large numbers of building blocks and unbounded cluster sizes. These assumptions also led to the use of mass-action, mean field descriptions such as the well known Becker Doering equations.

In cellular biology, however, nucleation events often take place in confined spaces, with a finite number of components, so that discreteness and stochastic effects must be taken into account.

In this talk we examine finite sized homogeneous nucleation by considering a fully stochastic master equation, solved via Monte-Carlo simulations and via analytical insight. We find striking differences between the mean cluster sizes obtained from our discrete, stochastic treatment and those predicted by mean field treatments.

We also consider heterogeneous nucleation stochastic treatments, first passage time results and possible applications to prion unfolding and clustering dynamics.

Baldvin Einarsson

Noise Driven Solutions of Schooling Fish and A Cellular Automata Model for Biofilm Growth with Surface Flow

The talk will be in two parts. In the first part, we will describe the model from Birnir:2007 and some of the solutions. We will then show that the average velocity of the particles tends to zero under most conditions. We therefore propose to add noise to the model and investigate whether some of the described structure of the solutions is maintained.

For the second part, we describe a two dimensional cellular automata model for biofilm growth in a rectangular tube with nonlaminar surface flow. Nutrient levels and structure of the biofilm determine the probability of the following mechanisms: i) Cell division and spreading ii) Cell erosion due to sheer stress iii) Production of EPS (extracellular polymeric substances) iv) Influx of cells which adhere to the surface and biofilm. We describe these mechanisms and the numerical code used to simulate the model. We then show how the model reproduces biofilm development in the form of flat biofilms, ripples, streamers, towers, "mushroom" growth etc. Finally, we briefly describe the effect of rugosity and an extension to three dimensions.

Raluca Eftimie

The role of communication mechanisms on the movement of self-organised aggregations

The formation, persistence and movement of self-organised biological aggregations are mediated by signals (e.g., visual, acoustic or chemical) that organisms use to communicate with each other. To investigate the effect that different communication mechanisms have on the movement of biological aggregations, we use a class of nonlocal hyperbolic models that incorporate social interactions. We calculate analytically the maximum speed for left-moving and right-moving groups, and show numerically that the travelling pulses exhibited by the nonlocal hyperbolic models actually travel at this maximum speed. We also show that the way organisms communicate with each other influences the magnitude of the speed of newly formed groups. However, it does not influence the magnitude of the speed of groups that have travelled for a long time. Finally, we discuss the role of communication mechanisms and social interactions on the choice of movement direction of travelling groups.

Radek Erban

Modelling multi-particle systems in cellular and molecular biology

I will discuss methods for spatio-temporal modelling in cellular and molecular biology. Three classes of models will be analysed in detail: (i) microscopic (molecular-based) models which are based on the simulation of trajectories of individual molecules and their reactive collisions; (ii) mesoscopic (lattice-based) models which divide the computational domain into a finite number of compartments and simulate the time evolution of the numbers of molecules in each compartment; and (iii) hybrid (multiscale) algorithms which use models with a different level of detail in different parts of the computational domain. All three classes of models (i)-(iii) are stochastic. The connections between these models and the deterministic models (based on reaction-diffusion-advection partial differential equations) will be presented. If time permits, I will also discuss hybrid modelling of chemotaxis where an individual-based model of cells is coupled with PDEs for extracellular chemical signals. Travelling waves in these hybrid models will be investigated.

Carlos Escudero Liébana

Kinetic theory two-species coagulation

We will present results concerning the stochastic process of two-species coagulation. Analytically, we derive a kinetic theory that approximately describes the process dynamics and determine its asymptotic behavior. We compare our analytical results with direct numerical simulations of the stochastic process and both corroborate its predictions and check its limitations.

Klemens Fellner

Aggregation patterns in non-local equations: discrete stochastic and continuum modelling.

Non-local evolution equations featuring interaction of individuals due to a repulsive-aggregative potential are observed to produce a rich dynamical behaviour, which leads to a multitude of stationary pattern. For interaction potential with suitable attractive singularities convergence to measure solutions is deduced from a gradient flow structure in Wasserstein metric. Alternatively propagate singular repulsive interaction potential regular solutions. However, the case of interaction potential with both singular and repulsive singularities remains an open problem, for which we present an interesting comparison of numerical results with a stochastic lattice model.

Joint work with E. Hackett-Jones, B. Hughes, K. Landman, University of Melbourne.

Razvan Fetecau

Swarm dynamics and equilibria for a nonlocal aggregation model

We consider the aggregation equation $\rho_t - \nabla \cdot (\rho \nabla K * \rho) = 0$ in $\mathbb{R}^n$ where the interaction potential K models short-range repulsion and long-range attraction. We study a family of interaction potentials with repulsion given by a Newtonian potential and attraction in the form of a power law. We show global well-posedness of solutions and investigate analytically and numerically the equilibria and their global stability. The equilibria have biologically relevant features, such as finite densities and compact support with sharp boundaries. This is joint work with Yanghong Huang and Theodore Kolokolnikov.

Eric Forgoston

Coherent Pattern Prediction in Swarms of Delay-Coupled Agents

We consider a general swarm model of self-propelling agents interacting through a pair-wise potential in the presence of noise and communication time delay. In previous work [Phys. Rev. E 77, 035203(R) (2008)], we showed that a communication time delay in the swarm induces a bifurcation that depends on the size of the coupling amplitude. We extend these results by unfolding the bifurcation structure of the mean field approximation. Our analysis reveals a direct correspondence between the different dynamical behaviors found in different regions of the coupling-time delay plane with the different classes of simulated coherent swarm patterns. We derive the spatio-temporal scales of these swarm structures, and also demonstrate how the complicated interplay of coupling strength, time delay, noise intensity, and choice of initial conditions can affect the swarm.

Amic Frouvelle

Macroscopic limits of a system of self-propelled particles with phase transition.

The Vicsek model, describing alignment and self-organisation in large systems of self-propelled particles, such as fish schools or flocks of birds, has attracted a lot of attention with respect to its simplicity and its ability to reproduce complex phenomena. We consider here a time-continuous version of this model, in the spirit of the one proposed by P. Degond and S. Motsch, but where the rate of alignment is proportional to the mean speed of the neighboring particles. In the hydrodynamic limit, this model undergoes a phase transition phenomenon between a disordered and an ordered phase, when the local density crosses a threshold value. We present the two different macroscopic limits we can obtain under and over this threshold, namely a nonlinear diffusion equation for the density, and a first-order non-conservative hydrodynamic system of evolution equations for the local density and orientation.

Joint work with Pierre Degond and Jian-Guo Liu.

Veysel Gazi

On the Stability of Swarms with Second Order Agent Dynamics

In this talk we discuss the overall dynamics and stability of swarms composed of agents with point mass second order dynamics. The inter-agent interactions in the individual based swarm model are provided with artificial potential functions. We discuss the aggregation, social foraging, and formation control problems.

Seung-Yeal Ha

Asymptotic formation of multi-clusters for the Cucker-Smale and Kuramoto models

In this talk, we will discuss asymptotic dynamics of two prototype models for flocking and synchronization, namely the Cucker-Smale and Kuramoto models. For these two models, we will present the asymptotic formation of multi-clusters. In previous literature for the Cucker-Smale model, the mono-cluster formation(global flocking) has been extensively studied by many researchers. In this talk, we will derive sufficient conditions for the multi-cluster formation to the particle and kinetic Cucker-Samle and Kuramoto models.

Jan Haskovec

Synchronization and aggregation emerging from random processes

We present two individual based models where social phenomena emerge purely from random behaviour of the agents, without introducing any deterministic "social force" that would push the system towards its organized phase. Instead, organization on the global level results merely from reducing the individual noise level in response to local organization, which is induced by stochastic fluctuations. The first model describes the recently experimentally observed collective motion of locust nymphs marching in a ring-shaped arena and is written in terms of coupled velocity jump processes. The second model was inspired by observations of aggregative behaviour of cockroach nymphs in homogeneous environments and is based on randomly moving particles with individual diffusivities depending on the perceived average population density in their neighbourhood. We show that both models have regimes leading to global self-organization of the group (synchronization and aggregation). Moreover, we derive the mean-field limits for both models, leading to PDEs with nonlocal nonlinearities, perform their mathematical analysis and present a few interesting numerical examples.

Yanghong Huang

Steady states and asymptotic limits of a nonlocal aggregation model

We consider the steady states of the aggregation equation $\rho_t = \nabla \cdot (\rho \nabla K * \rho)$, where the interaction potential K models short-range singular repulsion and long-range power-law attraction. We show that there exist unique radially symmetric equilibria supported on a ball. We perform asymptotic studies for the limiting cases when the exponent of the power-law attraction approaches infinity and a Newtonian singularity, respectively. Numerical simulations suggest that equilibria studied here are global attractors for the dynamics of the aggregation model.

Theodore Kolokolnikov

Asymptotics of complex patterns in an aggregation model with repulsive-attractive kernel

The aggregation model with short-range attraction and long-range repulsion can lead to very complex and intriguing patterns in two or three dimensions. Depending on the relative strengths of attraction and repulsion, a multitude of various patterns is observed, from nearly-constant density swarms to annular solutions, to complex spot patterns that look like "soccer balls". We show that many of these patterns can be understood in terms of stability and perturbations of "lower-dimensional" patterns. For example, spots arise as bifurcations of point clusters [delta concentrations]; annulus and various triangular shapes are perturbations of a ring. Asymptotic methods provide a powerful tool to describe the stability, shape and precise dimensions of these complex patterns.

Joint works with Bertozzi, von Brecht, Fetecau, Huang, Hui, Pavlovsky, Uminsky.

Thomas Boris Laurent

Aggregation via Newtonian potential and aggregation patches

We consider the motion of a density of particles $rho(x,t)$ by a velocity field $v(x,t)$ obtained by convolving the density of particles with the gradient of the Newtonian potential, that is $v = -\nabla N * \rho$. An important class of solutions are the ones where the particles are uniformly distributed on a time evolving domain. We refer to these solutions as aggregation patches, by analogy to the vortex patch solutions of the 2D incompressible Euler equations. Numerical simulations as well as some exact solutions show that the time evolving domain on which the patch is supported typically collapses on a complex skeleton of codimension one. We also show that going backward in time, any bounded compactly supported solution converges as t goes to minus infinity toward a spreading circular patch. We provide a rate of convergence which is sharp in 2D. This is a joint work with Bertozzi and Leger.

Joceline Lega

Coherent behaviors in groups of locally interacting particles

I will present results of molecular dynamics simulations of disks moving in a two-dimensional box and interacting through special collisions (J. Lega, SIAM J. Appl. Dyn. Sys. 10, 1213 -1231 (2011)). Because this work was motivated by the existence of complex behaviors in colonies of bacteria, the particles also reorient themselves at random times, thereby simulating bacterial tumbles and inputing energy into the system. I will show that at low packing fractions clusters dynamically form and break up and that, as the packing fraction increases, groups of increasingly larger size are observed, in which the particles move coherently. Such behaviors are markedly different from those observed in systems of particles interacting through elastic collisions. Time permitting, I will also present results on groups of locally interacting rods.

Doron Levy

Mathematical Models for Phototaxis

Certain organisms undergo phototaxis, that is they migrate toward light. In this talk we will discuss our recent results on modeling phototaxis in order to understand the functionality of the cell and how the motion of individual cells is translated into emerging patterns on macroscopic scales. This is a joint work with Amanda Galante, Susanne Wisen, Tiago Requeijo, and Devaki Bhaya.

Alan Lindsay

Quenching properties of a fourth order parabolic non-linear PDE in 2D geometries.

This talk will present some recent results on a singularity formation problem in a nonlinear fourth order PDE modelling a Micro-Elcetro Mechanical Systems (MEMS) Capacitor. The singularity observed is a divergence in the solution derivative, rather than the solution itself - a phenomenon known as quenching. The local structure of singularity is shown to be self-similar in nature. Additionally, the singularity is observed to form in multiple locations within the domain with these locations exhibiting an analyzable dependence on the model parameters and the geometry of the domain. We outline an asymptotic method which can predict the location(s) where singular solutions form based on the geometry of the domain and the parameters of the system. The theory is demonstrated on several examples. This is joint work with J. Lega (Arizona) and F.J. Sayas (Delaware).

Stephan Martin

Explicitly computable flock and mill states of self-propelled particles systems

We consider a self-propelled interacting particle system, which has been frequently used to model complex behavior of swarms such as fish schools or birds flocks. Typically, the model is equipped with the Morse potential and patterns such as aligned flocks and rotating mills emerge in particle simulations. To this day, these stationary states cannot be a priori computed, except for one-dimensional flocks. We present a class of interaction potentials, that we call Quasi-Morse potentials, which likewise generate flocks and rotating mills. However, their stationary states can be explicitly computed as (affine) linear combinations of up to three elementary functions, only whose scalar coefficients have to be obtained numerically. This can be achieved without simulating a time evolution. We present the formulae for the mill and flock solutions, verify our results by comparing to corresponding particle simulations and illustrate parameter dependencies.

Sebastien Motsch

A model of flocking with asymmetric interactions

In this talk, we introduce a model of flocking which aims at improving the popular Cucker-Smale (C-S) model. The C-S model relies on a simple rule: the closer two individuals are, the more they tend to align with each other. In the new model proposed, the strength of the interaction is also weighted by the density: the more an agent is surrounded, the less he will be influenced. As a consequence, interactions between agents are no longer symmetric.

To study the flocking behavior of this dynamics, we base our analysis on a Lˆ? approach rather than Lˆ2 approach. We find that the dynamics converges to a flock provided that the interaction function decays slowly enough.

Vladislav Panferov

Phase transitions in models of Vlasov-McKean type

I will discuss the problem of non-uniqueness and stability of steady states for equations of Vlasov-McKean type. These equations provide a mean-field description of a system of interacting diffusions; particularly we consider the problem with the spatial variable in a periodic box of size L and particles interacting through a pairwise potential V. If the Fourier transform of V has a negative minimum, the system has a critical threshold for the diffusion constant beyond which the trivial uniform steady state becomes unstable and the system experiences a phase transition. We show that for a large class of interactions, when the size of the domain is sufficiently large, the transition is always discontinuous and is characterized by coexistence of several stable states in a certain interval of parameter space.The transition is also shown to occur at a value of the diffusion constant strictly greater than the critical threshold. I will then briefly present the results of a numerical study on the character of phase transition in Vicsek like models of flocking, in which a similar discontinuous transition is observed.

Mark Pavlovski

Point cluster and spot patterns in an aggregation model with repulsive-attractive kernel

The aggregation model with short-range repulsion and long-range attraction generates diverse patterns in two or three dimensions. In this talk we will discuss stability of the point clusters as well as structure and stability of the spot patterns which arise as perturbations of the point clusters. This is a joint work with Theodore Kolokolnikov and Yanghong Huang.

Vakhtang Putkaradze

Molecular monolayers as interacting rolling balls: crystals, liquid and vapor

Molecular monolayers, especially water monolayers, are playing a crucial role in modern science and technology. In order to derive simplified models of monolayer dynamics, we consider the set of rolling self-interacting particles on a plane with an off-set center of mass and a non-isotropic inertia tensor. To connect with water monolayer dynamics, we assume the properties of the particles like mass, inertia tensor and dipole moment to be the same as water molecules. The perfect rolling constraint is considered as a simplified model of a very strong, but rapidly decaying bond with the surface. Since the rolling constraint is non-holonomic, it prevents the application of the standard tools of statistical mechanics: for example the system exhibits two temperatures – translational and rotational– for some degrees of freedom, and no temperature can be defined for other degrees of freedom.

In spite of apparent simplicity, the behavior of the system is surprisingly rich. We identify analogies with the regular water by defining crystalline, liquid and gas states, based on the specific energy of a particle. We show the existence and nonlinear stability of ordered lattice states,. We also investigate the effect of rolling on the disturbance propagation through a crystalline lattice, study the chaotic vibrations of the crystalline states and identify an interesting phase transition when the crystal is destroyed. We demonstrate that there are also relatively confined "droplet" states with the center of mass exhibiting seemingly random walk on the surface. Finally, we investigate the dynamics of disordered gas states and show that there is a surprising and robust linear connection between distributions of angular and linear velocity for both lattice and gas states, allowing to define the concept of temperature.

Finally, as a first step towards continuous theory, we develop a Vlasov-like kinetic theory for a gas of rolling balls. Using that framework, we show that the concept of momentum conservation cannot be borrowed from the classical fluids, and derive a set of alternative conservation laws for the system.

Gael Raoul

Nonlocal Aggregation Equations and Concentration Phenomena

Nonlocal aggregation equation appear in various fields of physics and biology. In many situations, the interaction potential presents a singularity at the origin. This singularity, which can be either attractive or repulsive, has a significant impact on the qualitative properties of solutions. In this talk, I will present a qualitative study of those qualitative properties in one and in several dimensions (although most questions remain open in the latter case). This work has been done in collaboration with Klemens Fellner, Daniel Balague, Jose Carrillo and Thomas Laurent.

Nancy Rodriguez

Hotspot invasion: Traveling Wave solutions for a Reaction-Diffusion system for crime patterns.

We analyze of a reaction-diffusion system of PDE's in order to obtain insight into propagation of crime patterns. This system of equations can be divided into three regimes, which lead to one, two, or three steady-states solutions. In this talk I will first discuss some preliminary results towards proving that indeed these are the only three possible regimes. Under the assumption that the parameters are heterogenous, I will discuss the condition that divide these regimes. Finally, I will discuss the invasion of hotspots via traveling wave solutions in one dimension.

Jesus Rosado

Kinetic Cucker-Smale model of collective behavior

We'll show the well posedness of the kinetic version of the Cucker-Smale model for flocking and prove that the unconditional flocking result that Cucker and Smale showed for the particle model also holds in the new framework. Then we'll discuss some new models that can be derived from it.

Michael Ward

Fekete Points, Narrow Escape, and Asymptotics of the Mean First Passage Time

The mean first passage time (MFPT) is calculated for a Brownian particle in a three-dimensional domain that contains N small non-overlapping absorbing windows on its boundary. The reciprocal of the MFPT of such narrow escape problems has wide applications in cellular biology where it may be used as an effective first order rate constant to describe, for example, the nuclear export of messenger RNA molecules through nuclear pores. In the asymptotic limit where the absorbing patches have small measure, the method of matched asymptotic expansions is used together with some detailed analytical properties of a surface Green's function to calculate a three-term asymptotic approximation for the MFPT for the unit sphere. The third term in this expansion depends explicitly on the spatial arrangement of the absorbing windows on the boundary of the sphere. The MFPT is minimized for particular trap configurations that minimize a certain discrete variational problem, which is closely related to the well-known optimization problem of determining the minimum energy configuration for N repelling Coulomb charges on the unit sphere. Finally, our three-term asymptotic expansion for the averaged MFPT is shown to be in very close agreement with full numerical results.

Joint work with: Alexei Cheviakov (U. Saskatchewan) and Ronny Straube (Max-Planck Institute; Magdeburg).

Bernt Wennberg

Propagation of chaos in biological swarm models

We consider two models of biological swarm behavior. In these models, pairs of particles interact to adjust their velocities one to each other. In the first process, called 'BDG', they join their average velocity up to some noise. In the second process, called 'CL', one of the two particles tries to join the other one's velocity. This paper establishes the master equations and BBGKY hierarchies of these two processes. It investigates the infinite particle limit of the hierarchies at large time- scale. It shows that the resulting kinetic hierarchy for the CL process does not satisfy chaos propagation. We present numerical simulations that indicate the same behavior for the BDG model. For the BDG model we also show, by explicit examples, that the stationary state may not be unique.

Authors: E. Carlen, R. Chatelin, P. Degond and B. Wennberg

Yao Yao

Degenerate diffusion with nonlocal aggregation: behavior of radial solutions

The Patlak-Keller-Segel (PKS) equation models the collective motion of cells which are attracted by a self-emitted chemical substance. While the global well-posedness and finite-time blow up criteria are well known, the asymptotic behaviors of solutions are not completely clear. In this talk I will present some results on the blow-up behavior and the asymptotic behavior of radial solutions. Numerically, we show that the solution exhibits three kinds of blow-up behavior: self-similar with no mass concentrated at the core, imploding shock solution and near-self-similar blow-up with a fixed amount of mass concentrated at the core. (joint work with A. Bertozzi) We also present some theoretical results concerning the asymptotic behavior of radial solutions when there is global existence. (joint work with I. Kim)