Mathematics 1000
Final Examination
Thursday, December 8, 2005
8:30 – 11:30 am

Dal ID Number: B00_________________ **Dal ID Card? Yes/No (circle one)**

Name:_________________________________ **Signature**_________________________
PRINT

Section: ____________________ and/or **Prof's Name:** ________________________

Show all work neatly and clearly. Please put boxes around your answers.
The reverse sides of the pages can be used for (unmarked) rough work.
Calculators, textbooks, notes etc. are not allowed.

Question	[Value]	[Mark]
1	10	
2	6	
3	8	
4	8	
5	8	
6	14	
7	7	
8	7	
9	12	
10	10	
11	10	
Total	100	

[MARKS]

1. Find the following limits.

[2] (a) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

[2] (b) $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 4} - 2}{x}$

[2] (c) $\lim\limits_{x \to \infty} \dfrac{3x^2 - x + 4}{2x^2 + 5x - 8}$

[2] (d) $\lim\limits_{\theta \to 0} \dfrac{\cos\theta - 1}{\theta}$

[2] (e) $\lim\limits_{x \to 0^+} x \ln x$

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[2] 2. (a) Complete the definitions:

(i) f(x) is <u>continuous</u> at $x = a$ if $\lim$

(ii) The <u>derivative</u> of a function $f(x)$ is $f'(x) = \lim$

[4] (b) (i) Complete the statement of the Intermediate Value Theorem: "Suppose f is continuous on the closed interval $[a,b]$ and let N be any number between $f(a)$ and $f(b)$, then there is a number c in (a,b) such that

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(ii) Use the Intermediate Value Theorem to show that there is a root of the equation $x^3+2x = x^2+1$ in the interval $(0,1)$.

[3] 3. (a) Find $\frac{dy}{dx}$ for the curve $x^3 + y^3 - 9xy = 0$.

[3] (b) Find the equation of the tangent to the curve in part (a) at the point (2,4). (Simplify as appropriate.)

[2] (c) For what values of x in the interval $[0,2\pi]$ does the graph of $f(x) = x + 2\cos x$ have a horizontal tangent?

4. Let $f(x) = (x^2-1)^{2/3}$.

[1] (a) What is the domain of $f(x)$?

[4] (b) Find the critical numbers of $f(x)$.

[3] (c) Show that $f''(x) = \frac{4}{9} \frac{x^2 - 3}{(x^2 - 1)^{4/3}}$.

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[4] 5. (a) Use logarithmic differentiation to find $\frac{dy}{dx}$ where $y = x^{\tan x}$.

[4] (b) Use the linearization $L(x) = f(a) + f'(a)(x-a)$ of $f(x) = \sqrt{x}$ at $a = 25$ to approximate $\sqrt{23}$.

6. Given $f(x) = xe^{-x^2/2}$, $f'(x) = (1-x^2)\, e^{-x^2/2}$, $f''(x) = x(x^2-3)\, e^{-x^2/2}$, fill in the spaces on the right below. If the answer is 'none', write 'none'. [Any rough work or table(s) can be done on the opposite (back) page. It will not be marked.]

[2] (a) State the domain of f and find any intercepts with the x-axis or y-axis.

Domain ______________________________
Coords. of intercept(s) ______________________

[4] (b) Complete:

(i) $\lim_{x \to \infty} xe^{-x^2/2} =$

(ii) $\lim_{x \to -\infty} xe^{-x^2/2} =$

(iii) Equation(s) of horizontal asymptote(s):___________________________

(iv) Equation(s) of vertical asymptote(s): ___________________________

[4] (c) Use f' given above to find:

(i) critical point(s) of f _______________________________________

(ii) interval(s) where f is increasing: ______________________________

(iii) interval(s) where f is decreasing: ______________________________

(iv) local minimum max/min (specify) point(s) (x,y) of f ________________

[3] (d) Use f" given above to find any

(i) interval(s) where f is concave up: ______________________________
(ii) interval(s) where f is concave down: _____________________________

(iii) point(s) of inflection of f ____________________________________

[1] (e) Use the information in (a) – (d) to sketch the graph of y = f(x) .

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[7] **7.** A rocket that is launched vertically is tracked by a radar station located on the ground 3 km from the launch site. What is the vertical speed of the rocket at the instant that its distance from the radar station is 5 km and this distance is increasing at a rate of 5000 km/h ?

[7] 8. A cylindrical drum is made to hold exactly 1m^3 in its interior. Assume that the material for the top and the bottom costs \$20 per m^2, while that for the side costs \$10 per m^2. Determine the radius of the drum that minimizes the cost of the material used.

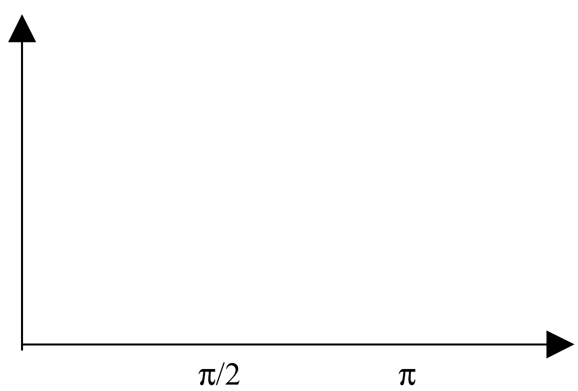
[6] 9. (a) Set up the Riemann sum to approximate the area under the curve $f(x) = \sin x$ between $x = 0$ and $x = \pi$ with $n = 4$ equal subintervals and with the sample points taken at the right endpoints. Complete the following, giving numerical values for any trigonometric expressions:

(i) Subinterval width $\Delta x =$

(ii) Sample points $x_i^* =$

(iii) Riemann sum is $R_4 =$

[2] (b) Illustrate your approximation in the diagram:



[2] (c) Show that $\int xe^x dx = xe^x - e^x + C$

[2] (d) Find the exact area under the curve $y = xe^x$ for $0 \le x \le 1$. [Hint: You can use the formula in part (c).]

10. Evaluate the following:

[2] (a) $\int_{1}^{2} \frac{t+5t^7}{t^3}\,dt$

[2] (b) $\int \frac{(\ln x)^3}{x}\,dx$

[2] (c) $\int_{0}^{1} \frac{1}{1+x^2}\,dx$

[2] (d) $\int_{0}^{1} e^{x^3} x^2\,dx$

[2] (e) Find $g'(x)$ if $g(x) = \int_{1}^{x} \frac{\cos t}{t}\,dt$.

[10] 11. Mark these statements True (T) or False (F). (Circle **T / F** as appropriate.)
If the answer is F , then give the correct answer in the space provided at the right.

(a) If $f(x) = \sin(\sin x)$, $f'(x) = \cos(\cos x)$ **T / F** ____________________

(b) If $f(x) = 5^x$, $f'(x) = x \ln 5$ **T / F** ____________________

(c) $\frac{d}{dx}(\sec^2 x - \tan^2 x) = 1$ **T / F** ____________________

(d) $\frac{d}{dx} \ln(e^x) = 1$ **T / F** ____________________

(e) $\frac{d}{dx}(\ln(\cos x)) = \tan x$ **T / F** ____________________

(f) $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$ **T / F** ____________________

(g) $\frac{d}{dx}\left(\frac{3}{10x^5}\right) = \frac{-3}{2x^6}$ **T / F** ____________________

(h) $\frac{d}{dx} \ln 3 = \frac{1}{3}$ **T / F** ____________________

(i) $\int_{-\pi/2}^{\pi/2} \cos x \, dx = 0$ **T / F** ____________________

(j) $\int_{-1}^{1} \frac{\sin x}{1 + x^4} dx = 0$ **T / F** ____________________

Use the space below for "rough work" for (a) – (j) . (It will not be marked.)