

# Review Questions for Midterm 1

- a) Complete the sentence: " $f(x)$  is continuous at  $x = a$  if ...".  
b) Consider the function  $f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x & \text{if } x \geq 1 \end{cases}$ . Sketch  $f$  and  $f'$ . Is  $f$  continuous at  $x = 1$ ? What about  $f'$ ?  
c) Consider the function  $f(x) = \begin{cases} 4 - x^2 & \text{if } x < 1 \\ x + a & \text{if } x \geq 1 \end{cases}$ . Find the value of  $a$  for which  $f(x)$  is continuous for all  $x$ . Sketch  $f$  and  $f'$ . Is  $f'$  continuous?
- Find these limits, or state why they don't exist:

$$\begin{array}{lll} \text{a) } \lim_{x \rightarrow 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} & \text{b) } \lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16} & \text{c) } \lim_{x \rightarrow 0} \frac{\sin(2x)}{3x} \\ \text{d) } \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} & \text{d) } \lim_{x \rightarrow \infty} \frac{\sqrt{4x^4 + 6}}{5x^2 + \sin(x)} & \text{e) } \lim_{x \rightarrow \infty} \sqrt{x^2 - 5x} - x \end{array}$$

- a) Complete the statement of the Intermediate Value Theorem: "If  $f(x)$  is continuous on  $[a, b]$  and  $N$  is any number between  $f(a)$  and  $f(b)$  then..."  
b) Show that  $2^x = x^3$  for some positive value of  $x$ .
- b) Use the definition of the derivative as a limit to find the derivative of  $f(x) = \frac{5}{x} - 1$  and  $f(x) = \sqrt{x} - 3x^2$ .
- Let  $f(x) = |x - 2|$ . Sketch  $f(x)$  and  $f'(x)$ . At which points is  $f$  continuous? What about  $f'$ ?
- a) Complete: "The graph of  $y = \frac{7-x}{(x-1)^2}$  has a vertical asymptote at  $x = \dots$  because .... It has a horizontal asymptote  $y = \dots$  because ..."  
b) Find

$$\lim_{x \rightarrow 3^+} \frac{x - 2}{(x - 3)(x - 4)} \quad \text{and} \quad \lim_{x \rightarrow 3^-} \frac{x - 2}{(x - 3)(x - 4)}.$$

- Find the derivatives of the following functions. Simplify as appropriate.

$$\begin{array}{lll} \text{a) } y = (\sqrt{x} - 3x^3)x^{-5} + e^3 & \text{b) } y = \sqrt{5x - 3} & \text{c) } y = \left(\frac{2x - 1}{3x + 1}\right)^4 \\ \text{d) } y = \sin^2 x & \text{e) } y = \sin(x^2) & \text{f) } y = \sin^2(x^2) & \text{g) } y = \sin(x) \cos(e^x) \end{array}$$

- Find the equation of the line tangent to  $y = 4x^2 - 5x - 6$  at  $(-1, 3)$ .
- Find the points on the curve  $y = \frac{x \exp(2x)}{3x - 1}$  where the tangent is horizontal.
- Newton's Law of Gravitation says that the magnitude  $F$  of the force exerted by a body of mass  $m$  on a body of mass  $M$  is

$$F = \frac{GmM}{r^2}$$

where  $G$  is the gravitational constant and  $r$  is the distance between the bodies.

(a) Find  $dF/dr$  and explain its meaning. What does the minus sign indicate?

(b) Suppose it is known that Earth attracts an object with a force that decreases at the rate of 2 N/km when  $r = 20,000$  km. How fast does this force change when  $r = 10,000$  km?

- For what values of  $x$  does the graph of  $f(x) = x + 2 \sin x$ ,  $0 \leq x \leq 2\pi$  have a horizontal tangent?

## More about the midterm:

Go through practice problems, AIM assignments and class notes.

Aim to write in clear sentences. Show all steps clearly.

**No calculators** of any kind will be allowed.

Put a box around your final answers.