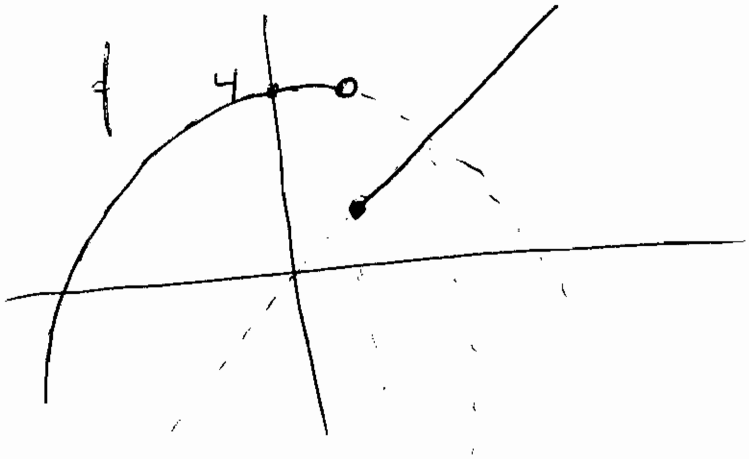
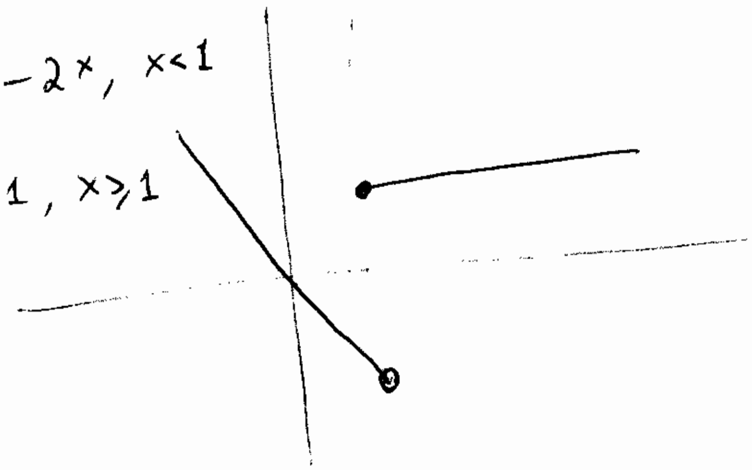
Sample Midterm 1 Answers

1 a) f is continuous at $x=a$ iff $f(a) = \lim_{x\to a} f(x)$.

1 b)



$f' = \begin{cases} -2x, & x<1 \\ 1, & x \geq 1 \end{cases}$



1c) $f(1^-) = 4 - 1^2 = 3$; $f(1^+) = 1 + a = 3$

$\Rightarrow \boxed{a = 2}$

2a) $-\frac{1}{9}$ 2b) $\frac{1}{8}$ 2c) $\frac{2}{3}$ 2d) 0 2e) $\frac{2}{5}$

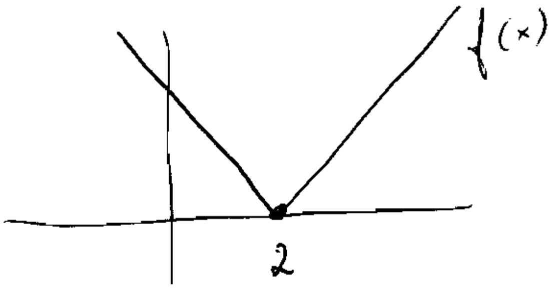
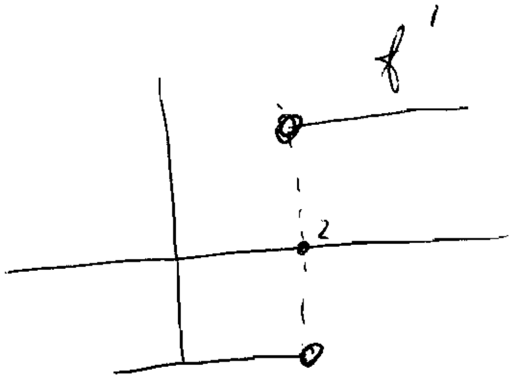
2f) $-\frac{5}{2}$

3b) Let $f(x) = 2^x - x^3$; $f(1) = 1$; $f(2) = -4$

$\Rightarrow f$ has a root between 1 and 2.

4a) $f(x) = \frac{5}{x} - 1$; $\frac{f(x+h) - f(x)}{h} = \frac{\frac{5}{x+h} - \frac{5}{x}}{h}$

$$= \frac{-5}{(x+h)\,x} \to \frac{5}{x^2}$$

5)  

f is cont. everywhere.

f' is not defined at $x=2 \Rightarrow$ Not cont. at $x=2$

6) $y \to \infty$ as $x \to 1$ and $y \to 0$ as $x \to \pm\infty$

7) a) $y = x^{-\frac{9}{2}} - 3x^{-2} + e^{3}$

$\Rightarrow y' = -\frac{9}{2} x^{-\frac{7}{2}} + 6x^{-3}$

7 b) $\frac{5}{2} \frac{1}{\sqrt{5x-3}}$ 7 c) $4\left(\frac{2x-1}{3x+1}\right)^3 \cdot \left(\frac{5}{(3x+1)^2}\right)$

7 d) $2 \sin x \cos x$ 7 e) $2x \cos(x^2)$

7 f) $2 \sin(x^2) \cos(x^2)\, 2x$

7 g) $\cos x \cos(e^x) - e^x \sin x \sin e^x$

8) $y - 3 = (-13)(x - (-1)) \Leftrightarrow y = -13x - 10$

9) $y' = \dfrac{(e^{2x} + 2xe^{2x})(3x-1) - 3xe^{2x}}{(3x-1)^2}$

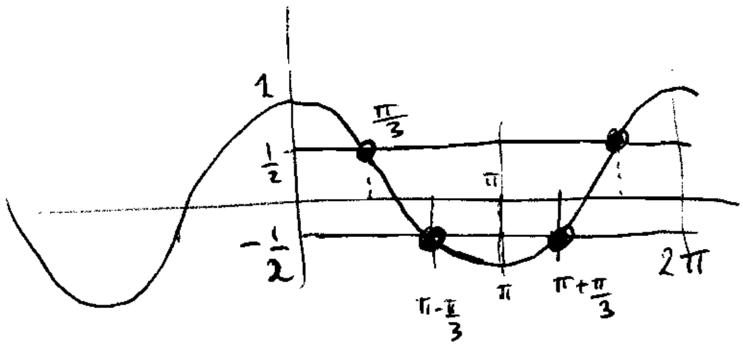
$= 0 \Leftrightarrow e^{2x}\left((1+2x)(3x-1) - 3x\right) = 0$

$\Leftrightarrow x = \dfrac{2 \pm \sqrt{28}}{12}$

10) 16 N/Km

11) $1 + 2\cos x = 0 \Rightarrow \cos x = -\frac{1}{2}$

$\Rightarrow$



Note that $\cos\frac{\pi}{3} = \frac{1}{2}$

Then from the graph of $\cos x$, we get that

$\cos\left(\pi \pm \frac{\pi}{3}\right) = -\frac{1}{2}$

$\Rightarrow \boxed{x = \pi \pm \frac{\pi}{3}}$