

## Review Questions for Midterm 2

1. Compute the derivatives of:  $f(x) = \ln\left(\frac{x}{1+x^2}\right)$ ;  $f(x) = x^{\ln x}$ ;  $f(x) = x \tan^{-1}(2x)$  [remark:  $\tan^{-1} \equiv \arctan$  ]
2. Consider a curve given implicitly by  $(x^2 + y^2)^2 = 4x^2y$ . (a) Find the slope of the tangent line at the point  $(1, 1)$ . (b) Estimate the value of  $y$  when  $x = 1.1$ .
3. Grain pouring from a chute at the rate of  $1/4 \text{ m}^3/\text{min}$  forms a conical pile whose height is always twice its radius. How fast is the height increasing when the pile is 2m high? (remark:  $V = \frac{1}{3}\pi r^2 h$ )
4. Sketch the graphs of

$$f(x) = x + \frac{1}{2x^2}; \quad f(x) = x^2 e^x \quad \text{and} \quad f(x) = \frac{x}{(1+x)^2}$$

. Label any max/min; asymptotes and inflection points. DO NOT use calculator/computer!!

5. Consider the function  $f(x)$  with the following properties:
  - $f(x)$  is an odd function [i.e.  $f(-x) = -f(x)$  ] which is continuous and differentiable for all  $x$ ;
  - $f(x) \rightarrow 0$  as  $x \rightarrow \infty$ ;
  - $f(x)$  is convex [concave up] for  $x > 1$  and is concave down for  $0 \leq x < 1$ .

Sketch how  $f(x)$  might look like. How many zeros, max/min and inflection points can it have?

6. Define "arcsec  $x$ "; Sketch its graph. What is its domain and range? Find its derivative.
7. Estimate  $\sqrt[3]{25}$  using linear approximation (hint: 25 is "close" to 27).
8. Your assignment is to make an ice cream cone out of a given fixed amount of dough. How should you do this in order to maximize the cone's volume? (remark: the volume of a cone is given in q.3).
9. Do problems 51 and 53 from Section 4.7 (page 339).
10. See the limit questions on AIM assignmnet #7.