Review midterm 2 Questions answers

1) (a) $\frac{1}{x} - \frac{2x}{1+x^2}$

(b) $f = e^{\ln(x^{\ln x})} = e^{(\ln x)^2}$

$\Rightarrow f' = 2(\ln x)\frac{1}{x}e^{(\ln x)^2}$

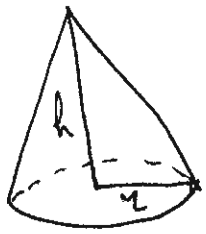
2) $2(x^2+y^2)(2x+2yy') = 4(2xy+x^2y')$

at $x=1$, $y=1 \Rightarrow$

$\cancel{4}(\cancel{2}+2y') = \cancel{4}(\cancel{2}+y')$

$\Rightarrow \boxed{y'=0}$

$y(1.1) \approx y(1) + 0.1\, y'(1) \approx 1$.

3) 

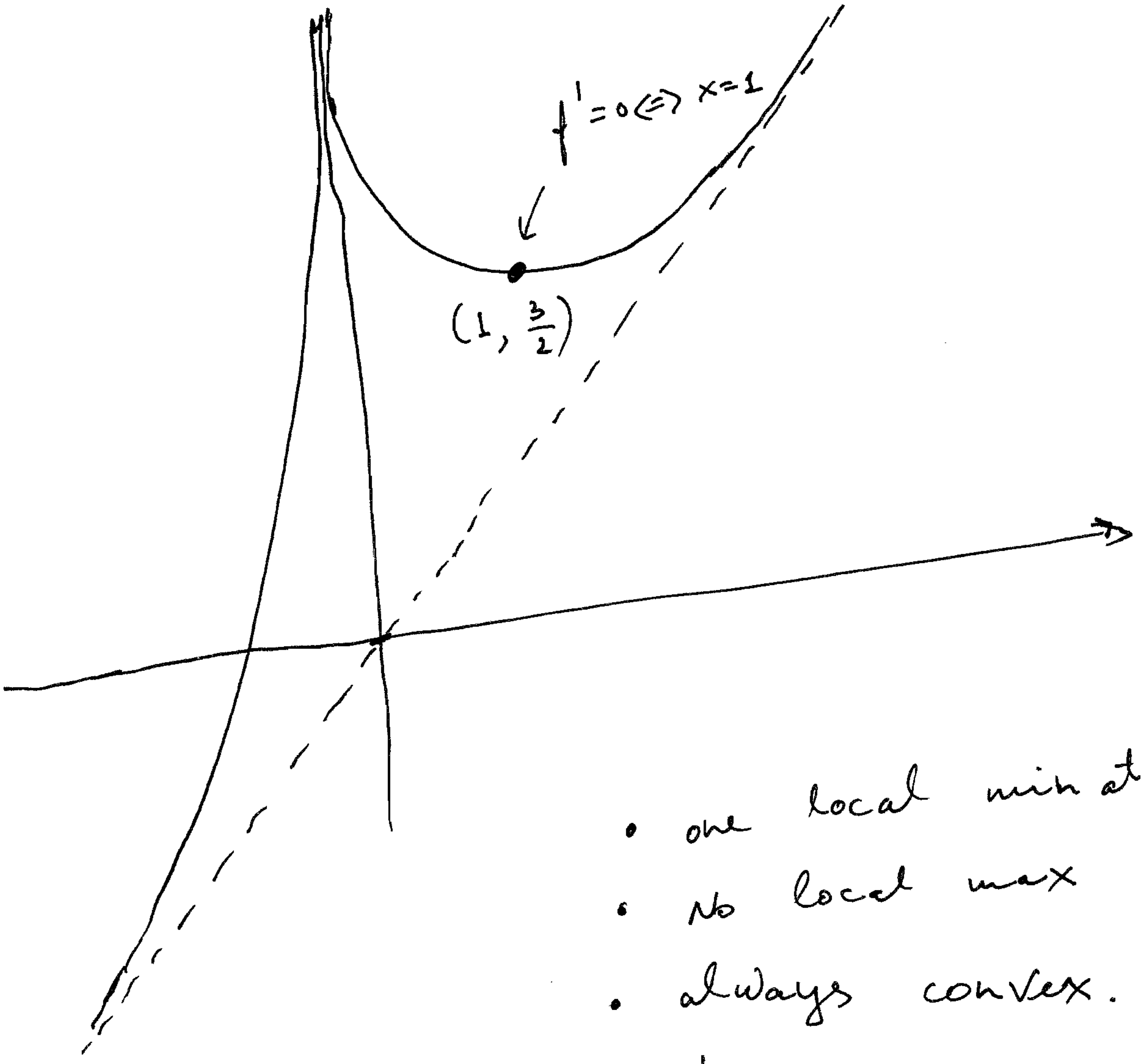
$h = 2r \Rightarrow r = \frac{h}{2}$

$V = \frac{\pi}{3}hr^2 = \frac{\pi}{12}h^3$

$\frac{1}{4} = \frac{dV}{dt} = \frac{\pi}{4}\underbrace{h^2}_{2^2}\frac{dh}{dt} \Rightarrow \boxed{\frac{dh}{dt} = \frac{1}{4\pi}}$

4a) $f = x + \frac{1}{2x^2}$; $f' = 1 - \frac{1}{x^3}$;

$f'' = \frac{3}{x^4} > 0$ for all x



- one local min at $x = 1$
- No local max
- always convex.
- $f(x) \sim x$ as $x \to \pm\infty$

4 b) $f(x) = x^2 e^x \qquad f'(x) = (2x + x^2)\, e^x$ 3

$f''(x) = (x^2 + 4x + 2)\, e^x$.

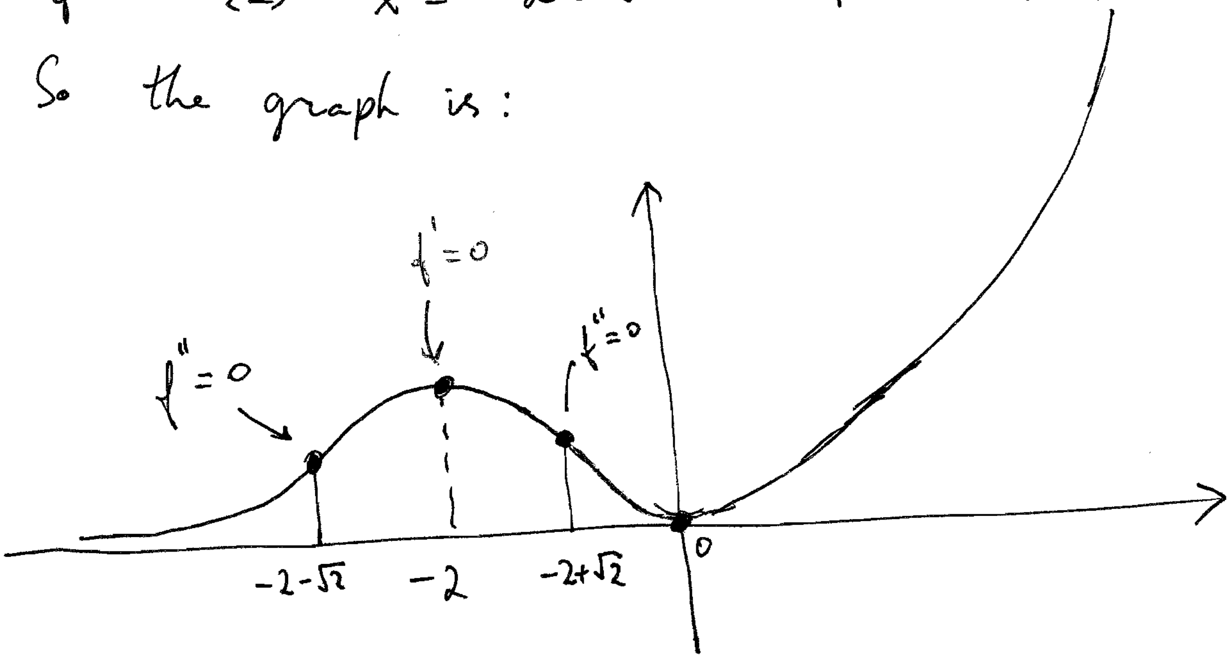
- $f' = 0 \iff x = 0$ or $x = -2$
- $f \to \infty$ as $x \to \infty$; $f \to 0$ as $x \to -\infty$

$\Rightarrow$ $x = 0$ is a local min;

$x = -2$ is a local max.

$f'' = 0 \iff x = -2 \pm \sqrt{2}$ (inflection pts)

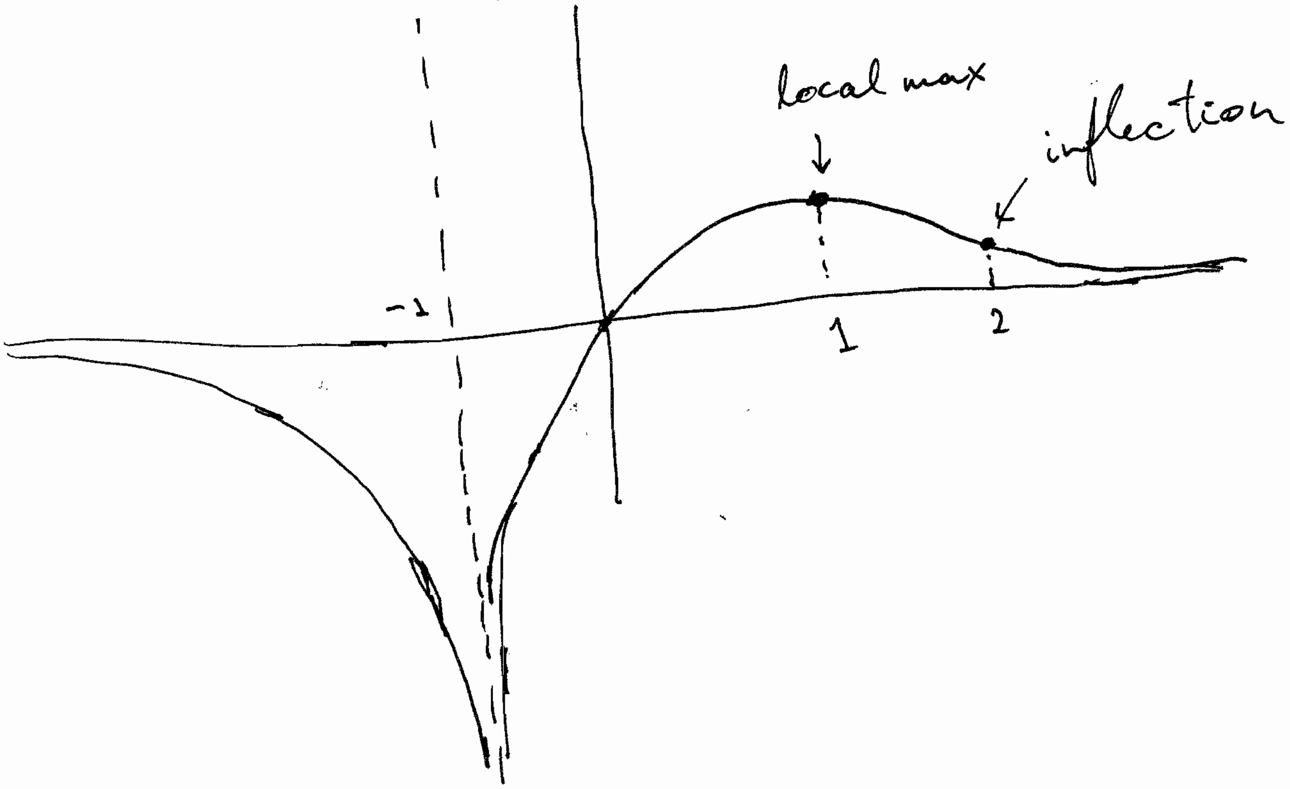
So the graph is:



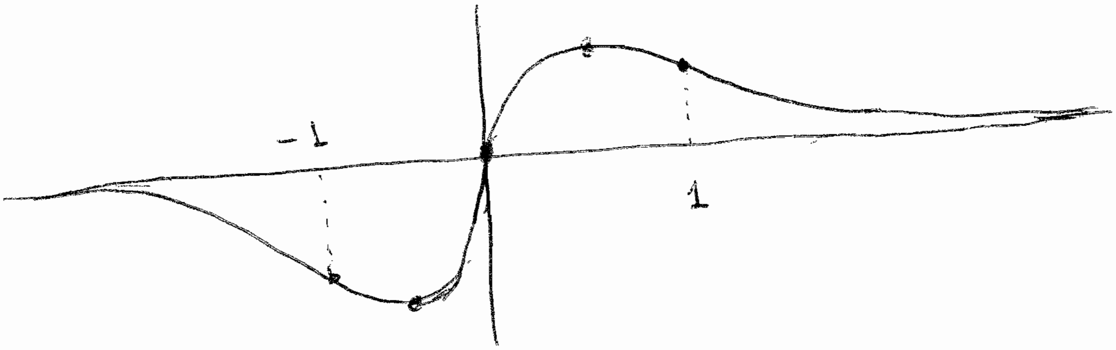
4c) $f(x) = \dfrac{x}{(1+x)^2}$

- $f \to 0$ as $x \to \pm\infty$; $f = 0 \Leftrightarrow x = 0$
- $f \to \cancel{\pm}\infty$ as $x \to -1^{\pm}$
- $f' = \dfrac{1-x}{(1+x)^3}$, $f'' = 2\dfrac{(x-2)}{(1+x^3)}$

$\Rightarrow$ max or min at $x = 1$;
inflection at $x = 2$.

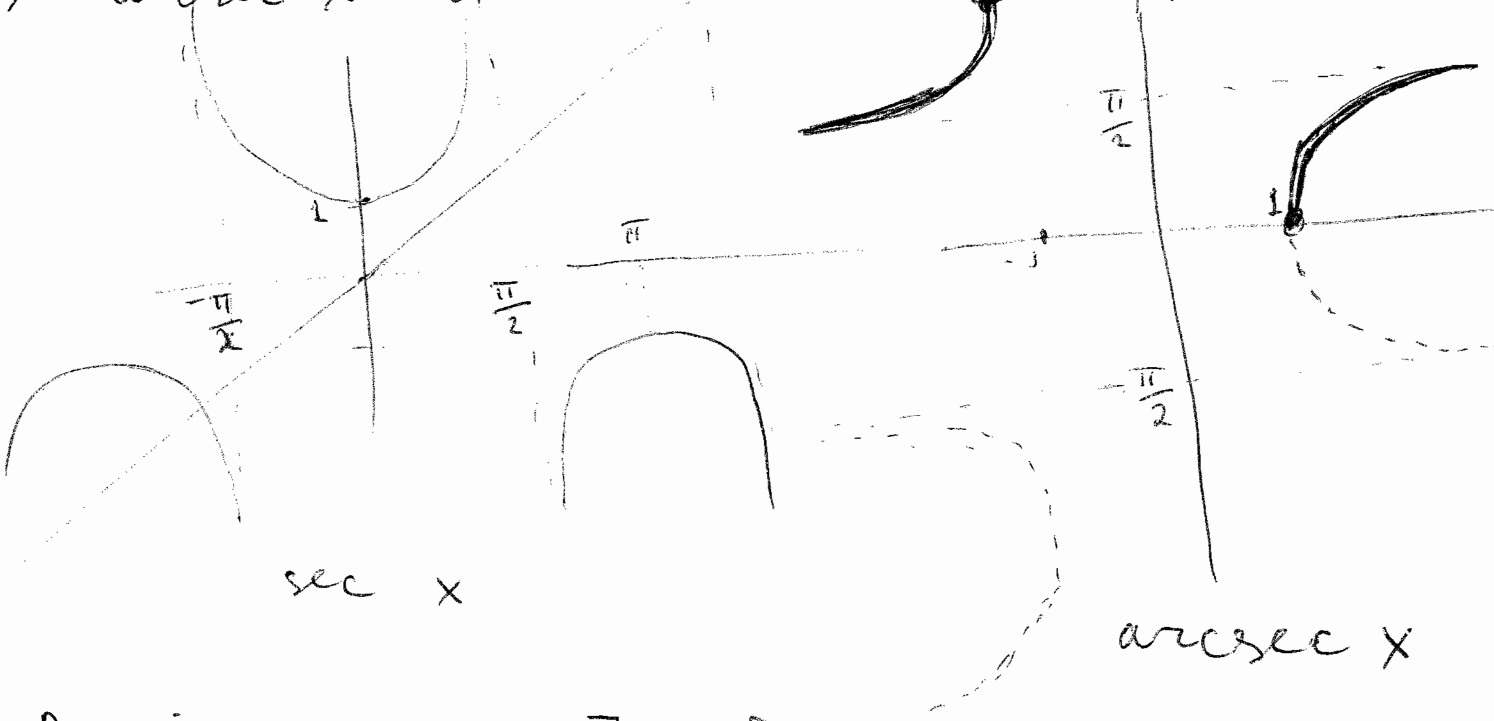


5)



- f has one zero at origin;
- 3 inflection points at x= -1, 0, 1
- one max between [0, 1]
- one min between [-1, 0]

6) arcsec x is the inverse of sec $x = \frac{1}{\cos x}$



<u>Domain:</u> $(-\infty, -1] \cup [1, +\infty)$

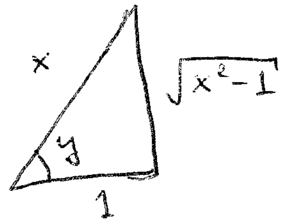
<u>Range:</u> $(\frac{\pi}{2}, \pi] \cup [0, \frac{\pi}{2})$

Derivative: $y = \operatorname{arcsec} x \Rightarrow \sec y = x$

$$\Rightarrow \cos y = \frac{1}{x}$$

$$\Rightarrow -y' \sin y = -\frac{1}{x^2}$$

$$y' = \frac{1}{x^2 \sin y}$$



$\cos y = \frac{1}{x}$

$\Rightarrow \sin y = \frac{\sqrt{x^2-1}}{x}$

$$\Rightarrow \boxed{y' = \frac{1}{x\sqrt{x^2-1}}}$$

7) Let $f(x) = x^{\frac{1}{3}}$; then $f' = \frac{1}{3} x^{-\frac{2}{3}}$;

$$f(27+h) \cong f(27) + h f'(27)$$

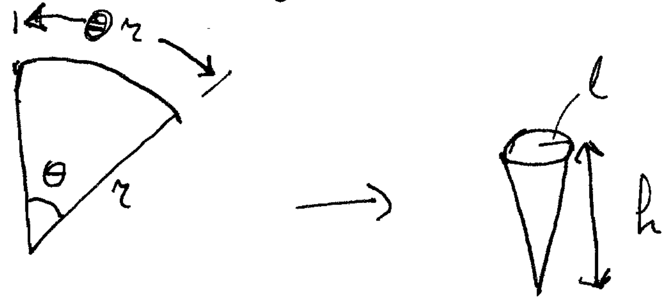
$$= 3 + \frac{h}{27}$$

Taking $h = -2$ we get $\sqrt[3]{25} = 3 - \frac{2}{27}$.

$$= 2.9592$$

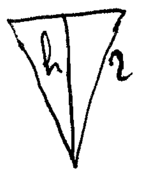
[calculator gives 2.9240177, relative error < 0.01%]

The cone is made by taking a segment of a circle & gluing the ends together:



We have:

$$A = \frac{\theta}{2} r^2$$



$$V = \frac{\pi}{3} l^2 h$$

$$2\pi l = \theta r$$

$$h^2 + l^2 = r^2$$

$$\Rightarrow \begin{cases} A = \frac{\theta}{2}(h^2+l^2) \Rightarrow \theta = \frac{2A}{h^2+l^2} \\ l^2 = \frac{\theta^2}{4\pi^2}(h^2+l^2) \end{cases}$$

$$\Rightarrow l^2 = \frac{A^2}{\pi^2(h^2+l^2)}$$

$$\Rightarrow h^2 = \frac{A^2}{\pi^2 l^2} - l^2$$

$$\boxed{V = \frac{\pi}{3} l^2 \sqrt{\left(\frac{A^2}{\pi^2 l^2} - l^2\right)}}$$

$$V(\ell)^2 = \left(\frac{\pi}{3}\right)^2\left(\ell^2 \frac{A^2}{\pi^2} - \ell^6\right)$$

$$= \left(\frac{\pi}{3}\right)^2\left(x\frac{A^2}{\pi^2} - x^3\right), \quad x = \ell^2$$

$$V' = 0 \iff (V^2)' = 0 \Rightarrow \frac{A^2}{\pi^2} - 3x^2 = 0, \quad x = \ell^2$$

$$\Rightarrow \boxed{\ell = 3^{-\frac{1}{4}} A^{\frac{1}{2}} \pi^{-\frac{1}{2}}}$$

$$\Rightarrow r = \frac{A^{\frac{1}{2}} 3^{\frac{1}{4}}}{\sqrt{\pi}}$$

$$h = A^{\frac{1}{2}} \pi^{-\frac{1}{2}} 3^{-\frac{1}{4}} \sqrt{2}$$

$$\boxed{\frac{h}{r} = \sqrt{2}}, \quad \boxed{\theta = \frac{2}{\sqrt{3}}\pi}.$$