Mathematics 1000
Test 2 – Practice Problems - November 2006

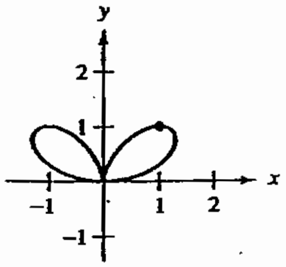
These questions indicate the style and level of difficulty of Test 2. The test itself will be shorter. The test is based on 3.6 - 4.7.

1. (i) $f(x) = \ln\left(\frac{x}{1+x^2}\right)$. Show that $f'(x) = \frac{1-x^2}{x(1+x^2)}$.

   (ii) $y = x^{\ln x}$. Use logarithmic differentiation to find $\frac{dy}{dx}$.

   (iii) $f(x) = x\tan^{-1}x$. Find $f'(1)$.

2. Find the slope of the tangent line to the curve $(x^2 + y^2)^2 = 4x^2y$ at the point $(1,1)$.



3. Grain pouring from a chute at the rate of $\frac{1}{4}$ m^3/min forms a conical pile whose height is always twice its radius. How fast is the height increasing when the pile is 2m high? (Use the formula $V = \frac{1}{3}\pi r^2 h$.)

4. Find the absolute max/min values of $f(x) = x + \frac{1}{2x^2}$ on $[\frac{1}{2}, 3]$.

5. Find the absolute max/min values of $f(x) = \frac{x-2}{x+1}$ on $[0,9]$.

6. Find these limits:

   (i) $\lim_{x\to-\infty} x^2e^x$ (ii) $\lim_{x\to 0}\frac{5^x - 3^x}{x}$ (iii) $\lim_{x\to\infty} x^{1/x}$

7. Complete:

   (i) $\frac{d}{dx}\ln(3x) =$ (ii) $\frac{d}{dx}\ln(e^x) =$ (iii) $\frac{d}{dx}\sin^{-1}x =$

8. Find the inflection points on $f(x) = x^4 + 3x^3 - x + 2$. (The x-coordinate is sufficient.)

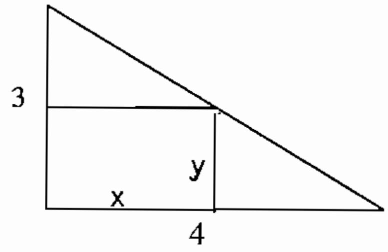
9. $f(x) = x^2e^x$. Find: (a) the intervals on which $f(x)$ is increasing or decreasing; (b) any local max/min points (x,y).

10. $f(x) = \dfrac{x}{(1+x)^2}$. (a) Show that $f'(x) = \dfrac{1-x}{(1+x)^3}$. (b) Find the intervals on which f is increasing or decreasing. (Make a table.) (c) Find any local max/min points.

11. Use implicit differentiation to find $\dfrac{dy}{dx}$ for $y = \sec^{-1}x$ $\left(0 \le y < \dfrac{\pi}{2} \text{ or } \pi \le y < \dfrac{3\pi}{2}\right)$.

12. Use the linearization formula $L(x) = f(a) + f'(a)(x-a)$ with $f(x) = \sqrt[3]{x}$, $a = 27$ to approximate $\sqrt[3]{25}$.

13. Set up this problem in the style of #4 above, but do not proceed further.
Find the area of the largest rectangle that can be inscribed in a triangle with legs 4 cm and 3 cm if two sides of the triangle lie along the legs.



(Find the area as $A(x)$ and state the domain.)

Answers

1. (ii) $x^{\ln x}\left(\dfrac{2\ln x}{x}\right)$ (iii) $\dfrac{\pi}{4}+\dfrac{1}{2}$

2. 0

3. $\dfrac{1}{4\pi}$ m/min

4. Max: $f(3) = \frac{55}{18}$, Min: $f(1) = \frac{3}{2}$

5. Max: $f(9) = \frac{7}{10}$, Min: $f(0) = -2$

6. (i) 0 (ii) $\ln\dfrac{5}{3}$ (iii) 1

7. (i) $\dfrac{1}{x}$ (ii) 1 (iii) $\dfrac{1}{\sqrt{1-x^2}}$

8. $x = 0$, $-\frac{3}{2}$

9. (a) incr $(-\infty,-2)$ and $(0,+\infty)$, decr $(-2,0)$ (b) $(-2,\frac{4}{e^2})$ max, $(0,0)$ min

10. Decr. on $(-\infty,-1)$, $(1,\infty)$; incr. on $(-1,1)$. Max at $(1,\frac{1}{4})$.

11. $\dfrac{1}{x\sqrt{x^2-1}}$

12. $\dfrac{79}{27}$

13. Find the absolute maximum value of $A(x) = \dfrac{12x-3x^2}{4}$ on $[0,4]$.