Sample midterm (modified from Feb. 2006)

1. Set up, but do not evaluate, the integral representing the area A of the region bounded by $x = 2y^2$ and $x + y = 1$. Include a sketch of the region.

2. Set up, but do not evaluate, an integral giving the volume of the solid of revolution obtained by rotating the region bounded by $y = \sqrt{x}$, the line $y = 2$ and the y-axis about the x-axis. Include a diagram.

3. Set up, but do not evaluate, an integral giving the volume of the solid of revolution obtained by rotating the region bounded by $y = x^2, y = 0, x = 1, x = 2$ about the line $x = 4$. Include a diagram.

4. Evaluate the following integrals:

$$a) \int \left(\frac{1}{2x} + \sin(2x) + \sec^2(x) \right) dx$$

$$b) \int x\sqrt{1+x}dx$$

$$c) \int x \cos 5x \; dx$$

$$d) \int \frac{3x-8}{x^2-2x}dx$$

$$e) \int_0^{\pi} \sin^3 x \; dx$$

$$f) \int \frac{x^2}{(16-x^2)^{3/2}}dx$$

$$g) \int \frac{5x^2+3x-2}{x^3+2x^2}dx$$

$$i) \int \frac{dx}{\sqrt{x^2-6x+13}}$$

5. Determine whether the following integrals converge or diverge (you do not need to evaluate them!):

$$a) \int_1^{\infty} \frac{\sqrt{x}}{x^2+1}dx$$

$$b) \int_0^1 \frac{e^x}{1-x}dx$$

6. Suppose that $f(x) = \int_0^x \sqrt{(\sin(t)+1)^2 - 1}dt$. Find the arclength of $y = f(x)$, with $x \in [0, \pi]$.

7. a) Use the Trapezoid rule to estimate $\int_{-1}^{1} \frac{2}{1+x^2}dx$ using $n = 4$.

   b) How large should you choose n in (a) to get the error less than 0.1? Remark: if $f(x) = \frac{2}{1+x^2}$ then it is known that $|f''| < 4$ for $x \in [-1, 1]$. The error for the Trapezoid rule is bounded by $\frac{(b-a)^3 \max|f''(x)|}{12n^2}$.