MATH 1500, Homework 12

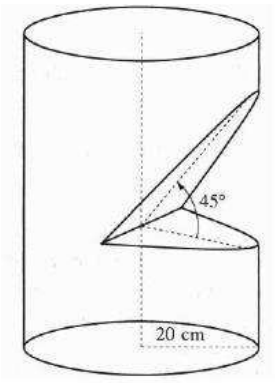
Due date: 3 February (Wed)

1. Let R be the region bounded by $y = x^2, y = \sqrt{x}$ between $x = 0$ and $x = 1$. Find the volume of the solid S obtained by rotating R around (a) the x axis; (b) the y axis.

2. Let R be a finite region bounded by $y = (2 - x)\,x$ and $y = 0,\ \ 0 \le x \le 2$. Find the volume of the solid obtained Find the volume of the solid S obtained by rotating R around (a) the x axis; (b) the y axis and (c) the line $x = -1$.

3. A solid has a circular base of radius r. All sections of the the solid perpendicular to a particular diameter of the base are squares. Sketch this solid and find its volume.

4. A 45^0 notch is cut to the centre of a cylindrical log having radius 20 cm as shown in the figure below. What volume of wood was removed from the log by cutting the notch?



5. Prove Pappus theorem for a special case where the region bounded by $0 \le y \le f(x)$, $a \le x \le b$ is rotated around the x-axis.

6. Consider the region between $y = x^2$, $y = x$, $0 \le x \le 1$.

   (a) Find the centroid of this region

   (b) Using Pappus theorem, find the volume of the solid obtained by rotating this region around the line $y = x$.

   (c) [BONUS] Do part (b) directly, without applying Pappus theorem.

7.

   (a) Find the center of mass of a triangular plate whose vertices have coordinates $(-a, 0)$, $(0,\ b)$, $(c,\ 0)$.

   (b) A piece of wire of uniform density is bent into a triangle whose vertices are given in part (a). Find its center of mass.

   (c) [BONUS] Under what geometric condition on a triangle do you get the same answer for part (a) and part (b)?

8. [BONUS] Find the center of mass of an eighth of a disk of unit radius, i.e. region bounded by the curves $y^2 + x^2 = 1,\ 0 \le x \le 1,\ 0 \le y \le x$. Hint: the center of mass of a quarter of a circle may be useful.

9. Determine the arclength of the curve $y = x^2$ with $x \in [0, 1]$.

10. Water from the ground level is being pumped into a conical tank [i.e. looks like a pylon] whose height is 1m and whose base has radius 1m. How much work must be done (against the gravity) to pump the water into the tank? Note that the water density is 1000kg/m3.

11. A spring has a natural length of 0.2 m. A 40 N force is required to stretch (and hold the spring) to a length of 0.3 m. How much work is done in stretching the spring from 0.35 m to 0.38 m? Remark: Use Hooke's law, there is a similar quesiton in the book.

12. A 10 meter cable weighs 40kg and hangs from the ceiling of a building without touching the floor. Determine the work that must be done to lift the bottom end of the chain all the way up until it touches the ceiling.