MATH 1500, Homework 3

<div align="center">Due: Wednesday, 7 October.</div>

1. Adams 1.4 (18, 19)

2. Adams, 1.4 (30, 31).

3. Adams 1.4 (32).

4. (From Adams p.93 q6) Consider the roots of the equation

$$ax^2 + 2x - 1 = 0 \tag{1}$$

   where $a > 0$. These are given by

$$r_+(a) = \frac{-1 + \sqrt{1+a}}{a}; \quad r_-(a) = \frac{-1 - \sqrt{1+a}}{a}.$$

   (a) Determine the limits

$$\lim_{a \to 0^+} r_+(a); \quad \lim_{a \to \infty} r_+(a).$$

   Show that $r_+(a) > 0$ for all $a > 0$. Based on this information, sketch a rough graph of r_+.

   (b) Determine the limits

$$\lim_{a \to 0^+} r_-(a); \quad \lim_{a \to \infty} r_-(a).$$

   Show that $r_-(a) > 0$ for all $a > 0$. Based on this information, sketch a rough graph of $r_-(a)$.

   (c) Suppose that $a = 0.01$. Estimate the two roots of (1) as best as you can but without using a calculator.

   (d) Suppose that $a = 100$. Estimate the two roots of (1) as best as you can but without using a calculator.

5. [BONUS] Adams, page 93 q.11.

6. [BONUS] Prove that if $g(x) = 1/f(x)$ then $g'(x) = -f'(x)/f(x)$.