Review Questions for Midterm 1

Midterm 1 will be held in-class on Wednesday, 14 October.

1. Find the following limits (some of them may be $\pm\infty$).

   a) $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{(x-3)}$ b) $\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16}$

   c) $\lim_{x\to 0} x^2 \sin\frac{1}{x}$ d) $\lim_{x\to\infty} \frac{\sqrt{4x^4+6}}{5x^2+\sin(x)}$ e) $\lim_{x\to\infty} \sqrt{x^2-5x}-x$

   f) $\lim_{x\to 3^-} \frac{x-2}{(x-3)(x-4)}$ g) $\lim_{x\to 3^+} \frac{x-2}{(x-3)(x-4)}$

2. (a) State the delta-epsilon definition of continuity.
   (b) Use delta-epsilon definition of continuity to show that the function $f(x)=x^2$ is continuous.
   (c) Use delta-epsilon definition of continuity to show that the function

   $$f(x)=\begin{cases} 1 & \text{if } x>0 \\ 0 & \text{if } x\le 0 \end{cases}$$

   is not continuous at $x=0$.

3. State the intermediate and maximum value theorems.

4. Show that $2^x = x^3$ for some positive value of x.

5. Consider the function $f(x)=\begin{cases} 4-x^2 & \text{if } x<1 \\ x+a & \text{if } x\ge 1 \end{cases}$. Find the value of a for which $f(x)$ is continuous for all x. Sketch f and f'. Is f' continuous?

6. Use the definition of the derivative as a limit to find the derivative of $f(x)=\frac{5}{x}$ and $f(x)=\sqrt{x}$.

7. Let $f(x)=|x-2|$. Sketch $f(x)$ and $f'(x)$. At which points is f continuous? What about f'?

8. Find the derivatives of the following functions. Simplify as appropriate.

   (a) $y=\left(\sqrt{x}-3x^3\right)x^{-5}+\cos(3)$ (b) $y=\sin(x^2\cos(x))$

   (c) $y=\sqrt{5x^2+3}$ (d) $y=\left(\frac{2x-1}{3x+1}\right)^4$ (e) $y=\frac{x-5}{(x-1)(x-4)}$.

9. Sketch the graph of functions (c), (d) and (e) from the preceeding question. Indicate any max/min and asymptotes. DO NOT use a computer.

10. (a) State the mean value theorem.
    (b) A function $f(x)$ satisfies $f(0)=0$, $f(1)=2$ and $f(2)=-1$. It is known that f is differentiable everywhere. Show that $f'(c)=0$ for some number c. Give complete justification, specifying any relevant theorems.
    (c) Prove that the function $f(x)=x^2-\cos x$ satisfies $f(x)=0$ for precisely two numbers x.